

Homework I:

I.1 #2, 10, 14, 20, 26, 30, 36, 50;

I.2 #2, 4, 12, 20;

I.3 #6, 10, 12, 14;

I.4 #2, 8, 10, 14, 18

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Exercises for Section 1.1

A. Write each of the following sets by listing their elements between braces.

$$\begin{array}{ccccccc} \dots & x=-2 & x=-1 & x=0 & x=1 & x=2 & \dots \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 2. \{3x+2 : x \in \mathbb{Z}\} & = & \{ \dots & -4, & -1, & 2, & 5, & 8, & \dots \} \end{array}$$

$$\begin{aligned} 10. \{x \in \mathbb{R} : \cos x = 1\} &= \{x \in \mathbb{R} : x \text{ is a multiple of } 2\pi\} \\ &= \{2\pi n : n \in \mathbb{Z}\} \\ &= \{\dots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \dots\} \\ \text{or } &\{0, \pm 2\pi, \pm 4\pi, \dots\} \end{aligned}$$

$$\begin{aligned} 14. \{5x : x \in \mathbb{Z}, |2x| \leq 8\} &= \{5x : |x| \leq 4\} \\ &= \{5x : x = -4, -3, -2, -1, 0, 1, 2, 3, 4\} \\ &= \{-20, -15, -10, -5, 0, 5, 10, 15, 20\} \\ \text{or } &\{0, \pm 5, \pm 10, \pm 15, \pm 20\} \end{aligned}$$

B. Write each of the following sets in set-builder notation.

$$\begin{aligned} 20. \{\dots, -8, -3, 2, 7, 12, 17, \dots\} &= \{5n+2 : n \in \mathbb{Z}\} \\ &\text{or } \{5n-3 : n \in \mathbb{Z}\} \\ &\text{or } \{5n+7 : n \in \mathbb{Z}\}, \text{ etc.} \end{aligned}$$

$$\begin{aligned} 26. \{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\} &= \{\dots, 3^{-3}, 3^{-2}, 3^{-1}, 3^0, 3^1, 3^2, 3^3, \dots\} \\ &= \{3^n : n \in \mathbb{Z}\} \end{aligned}$$

C. Find the following cardinalities.

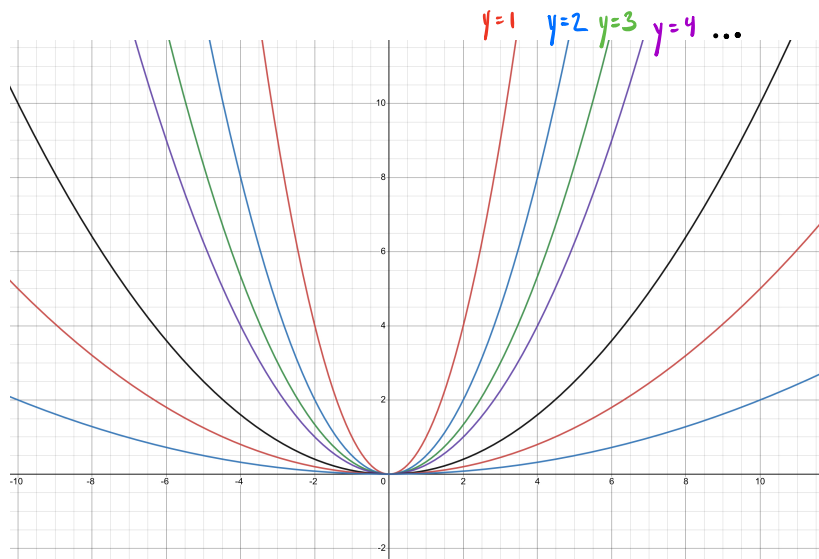
$$30. |\{\{1,4\}, a, b, \{3,4\}, \{\emptyset\}\}| = 5$$

Note that this set contains 3 elements that are sets,
and 2 of those elements are sets that contain a set.

$$36. |\{x \in \mathbb{N} : x^2 < 10\}| = |\{x \in \mathbb{N} : |x| < \sqrt{10} \approx 3.16\}| \\ = |\{1, 2, 3\}| = 3$$

D. Sketch the following sets of points in the x-y plane.

$$50. \{(x, \frac{x^2}{y}) : x \in \mathbb{R}, y \in \mathbb{N}\}$$



<https://www.desmos.com/calculator/ie5m9hptd3>

Exercises for Section 1.2

A. Write out the indicated sets by listing their elements between braces.

2. Suppose $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$.

- | | | | |
|------------------|------------------|-----------------------------|-----------------------------|
| (a) $A \times B$ | (c) $A \times A$ | (e) $A \times \emptyset$ | (g) $A \times (B \times B)$ |
| (b) $B \times A$ | (d) $B \times B$ | (f) $(A \times B) \times B$ | (h) $A \times B \times B$ |

$$(a) \{(\pi, 0), (\pi, 1), (e, 0), (e, 1), (0, 0), (0, 1)\}$$

$$(b) \{(0, \pi), (0, e), (0, 0), (1, \pi), (1, e), (1, 0)\}$$

$$(c) \{(\pi, \pi), (\pi, e), (\pi, 0), (e, \pi), (e, e), (e, 0), (0, \pi), (0, e), (0, 0)\}$$

$$(d) \{(0, 0), (0, 1), (1, 0), (1, 1)\}$$

(e) $\emptyset$: THERE ARE NO LISTS OF 2 THINGS SUCH THAT
THE SECOND THING IS AN ELEMENT OF THE EMPTY SET

(f)

$$\begin{array}{c} (\pi, 0) \quad (\pi, 1) \quad (e, 0) \quad (e, 1) \quad (0, 0) \quad (0, 1) \\ 0 \quad \left\{ ((\pi, 0), 0), (\pi, 1), 0), (e, 0), 0), (e, 1), 0), (0, 0), 0), (0, 1), 0), \right\} \\ 1 \quad \left\{ ((\pi, 0), 1), (\pi, 1), 1), (e, 0), 1), (e, 1), 1), (0, 0), 1), (0, 1), 1) \right\} \end{array}$$

(g)

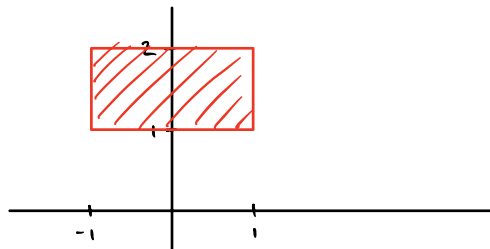
$$\begin{array}{c} \pi \quad e \quad o \\ (0, 0) \quad \left\{ (\pi, (0, 0)), (e, (0, 0)), (o, (0, 0)), \right. \\ (0, 1) \quad \left\{ (\pi, (0, 1)), (e, (0, 1)), (o, (0, 1)), \right. \\ (1, 0) \quad \left\{ (\pi, (1, 0)), (e, (1, 0)), (o, (1, 0)), \right. \\ (1, 1) \quad \left\{ (\pi, (1, 1)), (e, (1, 1)), (o, (1, 1)) \right\} \end{array}$$

$$(h) \quad \left\{ \begin{array}{l} (\pi, 0, 0), (e, 0, 0), (o, 0, 0), \\ (\pi, 0, 1), (e, 0, 1), (o, 0, 1), \\ (\pi, 1, 0), (e, 1, 0), (o, 1, 0), \\ (\pi, 1, 1), (e, 1, 1), (o, 1, 1) \end{array} \right\}$$

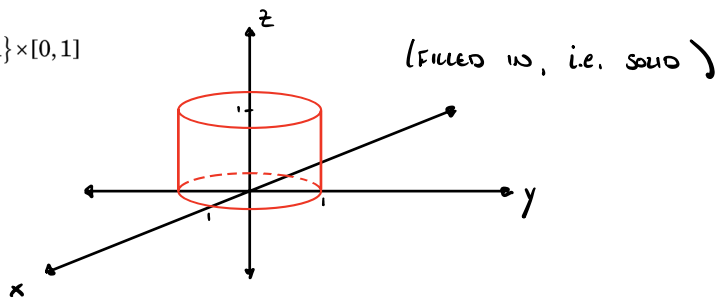
$$\begin{aligned} 4. \{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\} &= \{3, 4\} \times \{-5, 5\} \\ &= \{(3, -5), (3, 5), (4, -5), (4, 5)\} \end{aligned}$$

B. Sketch these Cartesian products on the x-y plane $\mathbb{R}^2$ (or $\mathbb{R}^3$ for the last two).

12. $[-1, 1] \times [1, 2]$



20. $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\} \times [0, 1]$



Exercises for Section 1.3

A. List all the subsets of the following sets.

6. $\{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}$

SUBSETS WITH 0 ELEMENTS

$\emptyset$,

SUBSETS WITH 1 ELEMENT

$\{\mathbb{R}\}, \{\mathbb{Q}\}, \{\mathbb{N}\},$

SUBSETS WITH 2 ELEMENTS

$\{\mathbb{R}, \mathbb{Q}\}, \{\mathbb{R}, \mathbb{N}\}, \{\mathbb{Q}, \mathbb{N}\},$

SUBSETS WITH 3 ELEMENTS

$\{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}$

B. Write out the following sets by listing their elements between braces.

10. $\{X \subseteq \mathbb{N} : |X| \leq 1\} = \{ \emptyset, \{1\}, \{2\}, \{3\}, \dots \}$

$\uparrow$
 SIZE 0 SIZE 1 "SINGLETONS"

12. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 1\} = \{\{3\}, \{2\}, \{a\}\}$

Note THAT SUBSETS ARE SETS, AND SO THE BRACKETS ARE NECESSARY.

C. Decide if the following statements are true or false. Explain.

14. $\mathbb{R}^2 \subseteq \mathbb{R}^3$

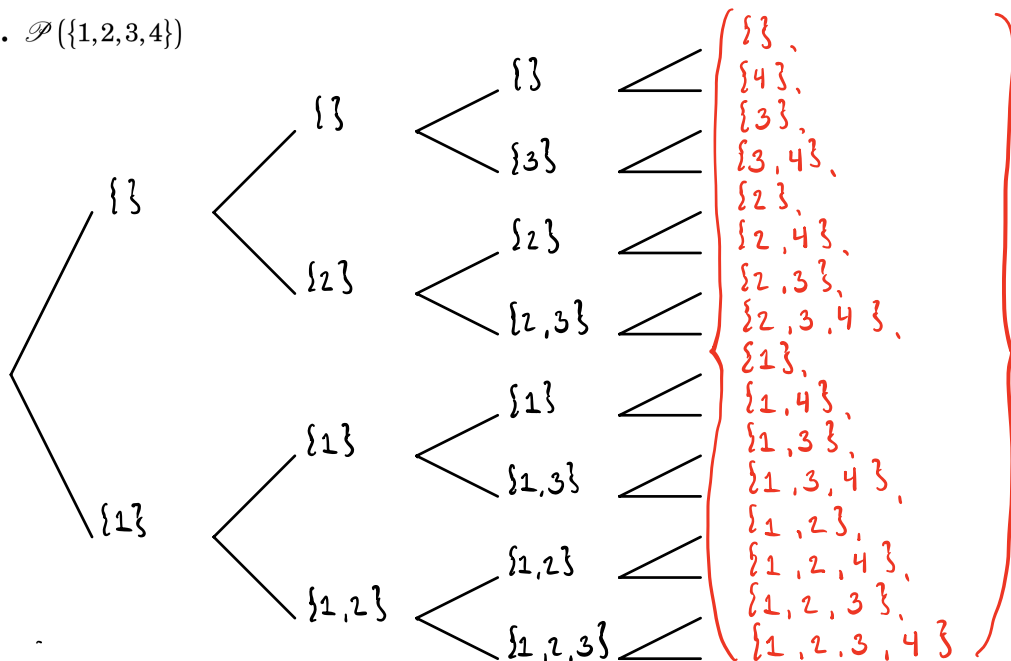
FALSE.

$\mathbb{R}^3$ CONTAINS ORDERED TUPLES. IT DOES NOT CONTAIN ANY ORDERED PAIRS.

Exercises for Section 1.4

A. Write the following sets by listing their elements between braces.

2. $\mathcal{P}(\{1,2,3,4\})$



8. $\mathcal{P}(\{1,2\} \times \{3\}) = \mathcal{P}(\{(1,3), (2,3)\})$
 $= \{\emptyset, \{(1,3)\}, \{(2,3)\}, \{(1,3), (2,3)\}\}$

10. $\{X \in \mathcal{P}(\{1,2,3\}) : |X| \leq 1\} = \{\emptyset, \{1\}, \{2\}, \{3\}\}$

B. Suppose that $|A| = m$ and $|B| = n$. Find the following cardinalities.

14. $|\mathcal{P}(\mathcal{P}(A))|$ Let $C = \mathcal{P}(A)$. Then $|C| = |\mathcal{P}(A)| = 2^{|A|} = 2^m$.
 Moreover, $|\mathcal{P}(\mathcal{P}(A))| = |\mathcal{P}(C)| = 2^{|C|} = 2^{2^m}$.

18. $|\mathcal{P}(A \times \mathcal{P}(B))|$ Since $|A| = m$ & $|\mathcal{P}(B)| = 2^{|B|} = 2^n$,

We have $|A \times \mathcal{P}(B)| = m 2^n$.

Thus, $|\mathcal{P}(A \times \mathcal{P}(B))| = 2^{|A \times \mathcal{P}(B)|} = 2^{m 2^n}$.