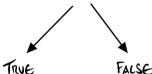
## CH. 6 PROOF BY COMMADICION

ex. Proposition. There are No instegens a, b e I such that 51a + 87b = 1.



FALSE THE STANGUENT MUST BE ONE OR THE OTHER.

PILLOF BY CONTINDICTION:

- Assume THE STAMEMENT IS FALSE.
- 2 FORM LOGICAL ANGUNELY TO CONCLUDE
- SOMETHING KNOWN TO BE FALSE (CONTRADICTION)

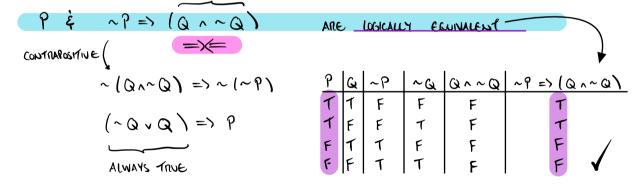
  THE ASSUMPTION CANNOT BE FALSE.

  THEREFORE ASSUMPTION IS TRUE.

Proof. (1) Assume there are integers a & b such that 51a + 87b = 1.

- (1) THEN 3(17a+29b)=1 AND 17a+29b=3. Since 17a+29b EZ, 17 FOLLOWS THAT 3 EZ.
- 3 This contradicts the Fact That  $\frac{1}{3}$  is not an integer. Therefore, our assumption that are integers  $a,b\in\mathbb{Z}$ . Such that 51a+67b=1 must be false.

CONTRADICTION: NEVER TRUE



Def:  $X \in \mathbb{R}$  is <u>national</u> if  $X = \frac{a}{b}$  for some  $a, b \in \mathbb{Z}$ .

THE SET OF NATIONAL NUMBERS IS

$$Q = \{ x \in \mathbb{R} : x = \frac{a}{b}, \text{ with } a, b \in \mathbb{Z} \}.$$

 $X \in \mathbb{R}$  is irrational if  $x \neq \frac{a}{b}$  for any  $a, b \in \mathbb{Z}$ 

## ex. THEOREM. TO IS IRRATIONAL.

Proof. Assume, For the sake of contradiction, that  $\sqrt{2} \in \mathbb{Q}$ . By Definition,  $\sqrt{2} = \frac{a}{b}$ .

Without loss of Generality, Let  $\frac{a}{b}$  be reduced. In Panticular,  $a \notin b$  are Not Both EVEN.

THEN  $\sqrt{2} = \frac{a}{b} \implies \sqrt{2}b = a \implies 2b^2 = a^2$ .
Thus  $a^2$  is even, AND SINCE THE PRODUCT OF TWO ODD NUMBERS IS DOD, IT MUST BE THAT a is even. Let a = 2x,  $x \in \mathbb{Z}$ .

THEN  $2b^2 = (2x)^2 = 4x^2 = b^2 = 2x^2$ . Thus  $b^2$  is EVED, AND so be MUST BE EVED. LET b = 2y,  $y \in \mathbb{Z}$ .

Thenefore Both a  $\xi$  b Are even, AND  $\frac{a}{b}$  is Not reduced. This contradicts the fact that  $\frac{a}{b}$  is Reduced.

THUS, OUR ASSUMPTION THAT  $\sqrt{2} \in \mathbb{Q}$  must be false. THAT IS,  $\sqrt{2}$  IS INNATIONAL.

## ex. THEREM. THERE ARE INFIDITELY MANY PRIME NUMBERS.

Prime numbers  $p_1, p_2, \ldots, p_n \in \mathbb{N}$ .

Set a= p. p. ... p. + 1 EN.

like all hadural numbers Gremer than 1, a has at least 1 prime divisor, say  $p_k$ . Then  $a = p_k \times \text{ for some } \times \in N$ .

WE HAVE PKX = P, P2 ... PK PK+1 ... Pn + 1

=) 
$$\frac{1}{p_{k}} = x - p_{1}p_{1}...p_{k-1}p_{k+1}...p_{n} \in \mathbb{Z}$$
.

But  $\frac{1}{p_K}$  cannot be an integer. =

## PROVING CONDITIONAL STATEMENTS BY CONTINDICTION

$$\frac{P_{ROP}:}{P_{ROP}:} P \Rightarrow Q.$$

$$\frac{P_{ROP}:}{P_{ROP}:} Assume \sim (P_{P})Q, i.e. P_{N} \sim Q.$$

$$\vdots$$

$$\Rightarrow \Leftarrow$$

**26.** If a and b are positive real numbers, then  $a + b \ge 2\sqrt{ab}$ .

Proof. (CONTRADICTION) ASSUME a,b Positive REAL NUMBERS  $\frac{1}{4}$   $a+b < 2\sqrt{ab}$ .

Then  $\sqrt{a}^2 - 2\sqrt{a}\sqrt{b} + \sqrt{b}^2 < 0$ That is,  $(\sqrt{a} - \sqrt{b})^2 < 0$ . => = > =

**9.** Suppose  $a, b \in \mathbb{R}$ . If a is rational and ab is irrational, then b is irrational.

PROOF: (CONTRADICTION) Assume  $a \in \mathbb{Q}$  and  $ab \notin \mathbb{Q}$  and  $b \in \mathbb{Q}$ .

BY DEFINITION,  $a = \frac{1}{g}$  and  $b = \frac{m}{n}$  For some  $p, g, m, n \in \mathbb{Z}$ .

Then  $ab = \frac{1}{g} \cdot \frac{m}{n} = \frac{pm}{gn}$ .

Since  $pm, gn \in \mathbb{Z}$ , We have  $ab \in \mathbb{Q} = \infty$