§ 2.4 BICONDITIONAL STATEMENTS

Def: THE CONVERSE OF THE CONDITIONAL STATEMENT $P \Rightarrow Q$ IS $Q \Rightarrow P$, WHICH IS DIFFERENT.

"WHAT ABOUT THE CONVERSE?"

ex. IF PETE IS A DOG THEN PETE IS A MAMMAL. (TRUE)

$\downarrow$ CONVERSE

IF PETE IS A MAMMAL THEN PETE IS A DOG. (FALSE)

ex. IF $x > 0$ THEN $-x < 0$. (TRUE)

$\downarrow$ CONVERSE

IF $-x < 0$ THEN $x > 0$. (TRUE)

Def: $P \Leftrightarrow Q \;=\; (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

"P ONLY IF Q AND P IF Q"

"P IF AND ONLY IF Q"

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$: $P \Leftrightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

"EITHER P & Q BOTH TRUE OR BOTH FALSE"

$\hookrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$

$$\left.\begin{array}{l} P \text{ if and only if } Q. \\ P \text{ is a necessary and sufficient condition for } Q. \\ \text{For } P \text{ it is necessary and sufficient that } Q. \\ P \text{ is equivalent to } Q. \\ \text{If } P\text{, then } Q\text{, and conversely.} \end{array}\right\} \quad P \Leftrightarrow Q$$

§2.5 TRUTH TABLES FOR STATEMENTS

Exercises for Section 2.5

INCLUDE "HELPER COLUMNS"

Write a truth table for the logical statements in problems 1–9:

1. $P \vee (Q \Rightarrow R)$ **4.** $\sim(P \vee Q) \vee (\sim P)$ **7.** $(P \wedge \sim P) \Rightarrow Q$

2. $(Q \vee R) \Leftrightarrow (R \wedge Q)$ **5.** $(P \wedge \sim P) \vee Q$ **8.** $P \vee (Q \wedge \sim R)$

3. $\sim(P \Rightarrow Q)$ **6.** $(P \wedge \sim P) \wedge Q$ **9.** $\sim(\sim P \vee \sim Q)$ *

10. Suppose the statement $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R and S. (This can be done without a truth table.)

11. Suppose P is false and that the statement $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true. Find the truth values of R and S. (This can be done without a truth table.)

* NOTE: $\sim$ BEHAVES LIKE $-$.

THAT IS $\sim P \wedge Q = (\sim P) \wedge Q \neq \sim(P \wedge Q)$

JUST AS $-a + b = (-a) + b \neq -(a+b)$

§2.6 LOGICAL EQUIVALENCE

DEF: TWO STATEMENTS ARE LOGICALLY EQUIVALENT IF THEIR TRUTH VALUES MATCH LINE-FOR-LINE IN A TRUTH TABLE.

EX. $(P \Rightarrow Q) = (\sim Q \Rightarrow \sim P)$

NO MATTER THE TRUTH VALUES OF P, Q!

NO MATTER THE VALUES OF x, y!

P	Q	$\sim P$	$\sim Q$	$(\sim Q) \Rightarrow (\sim P)$	$P \Rightarrow Q$
T	T	F	F	**T**	**T**
T	F	F	T	**F**	**F**
F	T	T	F	**T**	**T**
F	F	T	T	**T**	**T**

ANALOGY: $(x+y)^2 = x^2 + 2xy + y^2$

GIVEN TWO LOGICALLY EQUIVALENT STATEMENTS, ONE MAY BE EASIER TO PROVE!

A. Use truth tables to show that the following statements are logically equivalent.

1. $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$

~~**2.**~~ $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

3. $P \Rightarrow Q = (\sim P) \vee Q$

4. $\sim(P \vee Q) = (\sim P) \wedge (\sim Q)$

5. $\sim(P \vee Q \vee R) = (\sim P) \wedge (\sim Q) \wedge (\sim R)$

6. $\sim(P \wedge Q \wedge R) = (\sim P) \vee (\sim Q) \vee (\sim R)$

7. $P \Rightarrow Q = (P \wedge \sim Q) \Rightarrow (Q \wedge \sim Q)$

~~**8.**~~ $\sim P \Leftrightarrow Q = (P \Rightarrow \sim Q) \wedge (\sim Q \Rightarrow P)$

NOTE: DE MORGAN'S LAWS

- $\sim(P \vee Q) = (\sim P) \wedge (\sim Q)$
- $\sim(P \wedge Q) = (\sim P) \vee (\sim Q)$

ex.

Problem 4. Translate the following sentence into symbolic logic.

P: If it is raining, then it is windy and the sun is not shining.

Now, translate each of the following sentences into symbolic logic, and determine whether each sentence is logically equivalent to P or not.

(i) It is windy and not sunny only if it is raining.

(ii) Rain is a sufficient condition for wind with no sunshine.

(iii) It is not raining, if either the sun is shining or it is not windy.

(iv) Wind is a necessary condition for it to be rainy, and so is lack of sunshine.

(v) Either it is windy only if it is raining, or it is not sunny only if it is raining.

P: RAIN $\Rightarrow$ (WIND $\wedge \sim$ SUN)

(i) (WIND $\wedge \sim$ SUN) $\Rightarrow$ RAIN , NO

(ii) RAIN $\Rightarrow$ (WIND $\wedge \sim$ SUN) , YES

(iii) [(SUN $\wedge \sim$WIND) $\vee$ (WIND $\wedge \sim$SUN)] $\Rightarrow$ $\sim$RAIN, YES

(iv) (RAIN $\Rightarrow$ WIND) $\wedge$ (RAIN $\Rightarrow$ $\sim$SUN) , YES

(v) [(WIND $\Rightarrow$ RAIN) $\wedge \sim$($\sim$SUN $\Rightarrow$ RAIN)]

$\vee$ [($\sim$SUN $\Rightarrow$ RAIN) $\wedge \sim$(WIND $\Rightarrow$ RAIN)],

NO.