CH. 2 Logic

§2.1 STATEMENTS

Def: A **STATEMENT** IS A SENTENCE OR MATHEMATICAL EXPRESSION THAT IS EITHER TRUE OR FALSE.

IT MUST BE ONE, DEFINITIVELY.
IT CANNOT SOMETIMES BE TRUE, SOMETIMES FALSE.

ex. P : THE BRONX IS A BOROUGH OF NYC. ✓ TRUE

ex. Q : DON'T PARK IN THE BIKE LANE. ⊗ NOT DEFINITIVELY TRUE OR FALSE

ex. $R(x)$: IF $4x = 36$ THEN $x = 8$ ✓ FALSE

* **ex.** $S(x)$: $x^2 = 9$ ⊗ OPEN SENTENCE

ex. $S_1(x, y)$: $x^2 - y^2 = (x+y)(x-y)$ FOR ALL $x, y \in \mathbb{R}$. ✓ TRUE

ex $P(f, g)$: THE FUNCTION f IS THE DERIVATIVE OF THE FUNCTION g. ⊗ OPEN SENTENCE

* **Def:** A SENTENCE WHOSE TRUTH DEPENDS ON THE VALUE OF ONE OR MORE VARIABLE IS CALLED AN **OPEN SENTENCE**.

§2.2 AND, OR, NOT

WAYS TO COMBINE STATEMENTS TO MAKE NEW STATEMENTS

AND Let P : A TRIANGLE HAS 3 SIDES.
Q : A SQUARE IS A RECTANGLE.

Then $R = P \wedge Q$: A TRIANGLE HAS 3 SIDES AND A SQUARE IS A RECTANGLE.

IS THE STATEMENT THAT BOTH P & Q ARE TRUE.

TRUTH TABLE

LISTS ALL POSSIBLE TRUTH VALUES OF P, Q AND RESULTING TRUTH VALUE OF $P \wedge Q$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

ex.

$5 < 10 \wedge 11 > 10$ T
$8 > 6 \wedge 3 > 5$ F
$9 > 12 \wedge 8 < 12$ F
$1 > 2 \wedge 7 < 4$ F

OR $P \vee Q$: P OR Q

* IS THE STATEMENT THAT AT LEAST ONE OF THE STATEMENTS P, Q IS TRUE.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

ex.

$5 < 10 \wedge 11 > 10$ T
$8 > 6 \wedge 3 > 5$ T
$9 > 12 \wedge 8 < 12$ T
$1 > 2 \wedge 7 < 4$ F

ex. $(x \leq y) = (x < y) \vee (x = y)$

* TO STATE THAT EXACTLY ONE OF THE STATEMENTS P, Q IS TRUE, WE WRITE SOMETHING LIKE

- P OR Q, BUT NOT BOTH.
- EITHER P OR Q.
- EXACTLY ONE OF P OR Q.

NOT $\sim P$ = IT IS NOT TRUE THAT P. "NOT P"

ex. P : 15 IS A MULTIPLE OF 5.
$\sim P$: IT IS NOT TRUE THAT 15 IS A MULTIPLE OF 5.
(15 IS NOT A MULTIPLE OF 5.)

ex. Q : 10 IS ODD.
$\sim Q$: IT IS NOT TRUE THAT 10 IS ODD
(10 IS NOT ODD)
EVEN?

ex. Q : π IS ODD.
$\sim Q$: IT IS NOT TRUE THAT π IS ODD
(π IS NOT ODD)
EVEN? NO.

§2.3 CONDITIONAL STATEMENTS

$P \Rightarrow Q$ "IF P IS TRUE, THEN Q MUST ALSO BE TRUE."

"IF P THEN Q", "P IMPLIES Q"

"Q IS TRUE UNDER THE CONDITION THAT P IS TRUE."

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

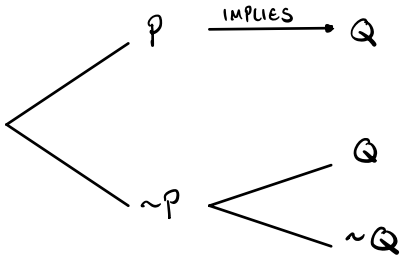
← THIS IS THE ONLY WAY FOR $P \Rightarrow Q$ TO BE FALSE.

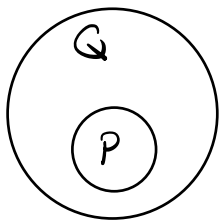
THINK P: I WIN THE LOTTERY.
Q: I TAKE YOU TO HAWAII

"P ONLY IF Q"

If P, then Q.
Q if P.
Q whenever P.
Q, provided that P.
Whenever P, then also Q.
P is a sufficient condition for Q.
For Q, it is sufficient that P.
Q is a necessary condition for P.
For P, it is necessary that Q.
P only if Q.

$\Big\} \; P \Rightarrow Q$





Without changing their meanings, convert each of the following sentences into a sentence having the form "*If P, then Q.*"

1. A matrix is invertible provided that its determinant is not zero.

2. For a function to be continuous, it is sufficient that it is differentiable.

3. For a function to be continuous, it is necessary that it is integrable.

IF A FUNC IS CONTINUOUS THEN IT IS INTEGRABLE.

4. A function is rational if it is a polynomial.

5. An integer is divisible by 8 only if it is divisible by 4.

6. Whenever a surface has only one side, it is non-orientable.

7. A series converges whenever it converges absolutely.

8. A geometric series with ratio r converges if $|r| < 1$.

9. A function is integrable provided the function is continuous.

10. The discriminant is negative only if the quadratic equation has no real solutions.

11. You fail only if you stop writing. (Ray Bradbury)