## §1.5 UNION, INTERSECTION, DIFFERENCE

**Definition 1.5** Suppose $A$ and $B$ are sets.
The **union** of $A$ and $B$ is the set $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
The **intersection** of $A$ and $B$ is the set $A \cap B = \{x : x \in A \text{ and } x \in B\}$.
The **difference** of $A$ and $B$ is the set $A - B = \{x : x \in A \text{ and } x \notin B\}$.

**ex.** LET $A = \{(x, \sin x) : x \in \mathbb{R}\}$
$B = \{(x, \csc x) : x \in \mathbb{R}\}$

FIND/SKETCH $A \cup B$, $A \cap B$, $A - B$, AND $B - A$.

**ex.** WHAT IS $\mathbb{R} - \mathbb{Q}$?

**ex.** LET $A = \{(x^2, x) : x \in \mathbb{R}\}$
$B = \mathbb{N} \times \mathbb{N}$

FIND/SKETCH $A \cup B$, $A \cap B$, $A - B$, AND $B - A$.

## §1.6 COMPLEMENTS

LET $A = \{n^2 : n \in \mathbb{N}\} = \{1, 4, 9, 16, \dots\}$

NAME AN OBJECT THAT IS NOT INSIDE $A$. TOO VAGUE!

ALMOST EVERY SET IS THOUGHT OF AS A SUBSET OF SOME LARGER SET THAT WE CALL THE UNIVERSAL SET $\mathcal{U}$.

**Def:** GIVEN A SET $A$ WITH UNIVERSAL SET $\mathcal{U}$,

THE COMPLEMENT OF $A$ IS $\overline{A} = \mathcal{U} - A$

**ex.** LET $A = \{(x, y) : 1 \le x^2 + y^2 \le 4\}$

SKETCH $A$ & $\overline{A}$

**ex.** WHAT IS $\overline{\mathbb{R}}$? DEPENDS ON $\mathcal{U}$: $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{C}$?

## §1.7 Venn Diagrams

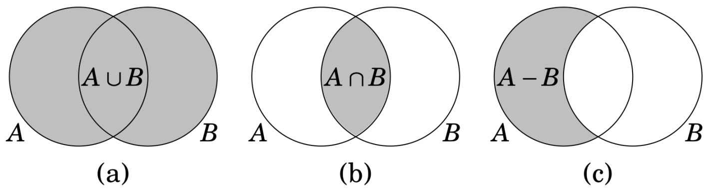
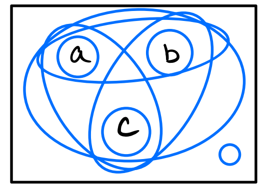
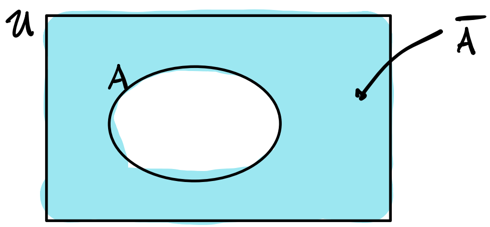



**Figure 1.7.** Venn diagrams for two sets

Review Power Sets:

$\mathcal{P}(\{a,b,c\})$



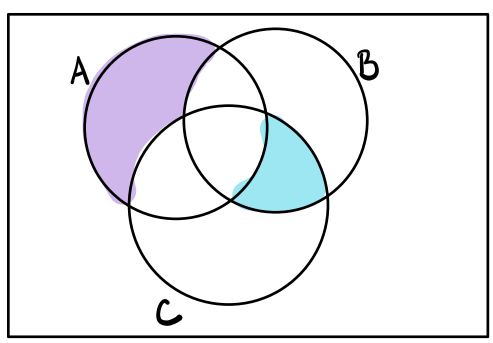




ex. Use Venn diagrams to verify De Morgan's Laws

(a) $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

(b) $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

With 3 sets:




$\bar{A} \cap B \cap C = (B \cap C) - A$

$A \cap \bar{B} \cap \bar{C} = (A - B) - C$

DIFFERENT!

$A - (B - C)$

**9.** Draw a Venn diagram for $(A \cap B) - C$.

**10.** Draw a Venn diagram for $(A - B) \cup C$.

**Important Points:**

- If an expression involving sets uses only $\cup$, then parentheses are optional.
- If an expression involving sets uses only $\cap$, then parentheses are optional.
- If an expression uses both $\cup$ and $\cap$, then parentheses are **essential**.

## § 1.8 Indexed Sets

Sets with subscripts

**Definition 1.7** Suppose $A_1, A_2, \ldots, A_n$ are sets. Then

$$A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n = \{x : x \in A_i \text{ for } \textit{at least one set } A_i, \text{ for } 1 \le i \le n\},$$

$$A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n = \{x : x \in A_i \text{ for } \textit{every} \text{ set } A_i, \text{ for } 1 \le i \le n\}.$$

Notation:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n \quad \text{and} \quad \bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n.$$

Even for an infinite # of indexed sets:

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots = \{x : x \in A_i \text{ for at least one set } A_i \text{ with } 1 \le i\}.$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots = \{x : x \in A_i \text{ for every set } A_i \text{ with } 1 \le i\}.$$

Alternatively, we may write $\displaystyle\bigcup_{i=1}^{3} A_i = \bigcup_{i \in \{1,2,3\}} A_i$,

or, if we set $I = \{1,2,3\}$, the $\displaystyle = \bigcup_{i \in I} A_i$

Here $i$ is the index & $I$ is called the index set (and $A_i$'s are indexed sets)

e.g. $\displaystyle\bigcap_{i=1}^{\infty} A_i = \bigcap_{i \in \mathbb{N}} A_i$, for example.

ex. Use interval notation to describe (a) $\displaystyle\bigcup_{n \in \mathbb{N}} \left[\frac{1}{n}, n\right]$ (b) $\displaystyle\bigcap_{n \in \mathbb{N}} \left[\frac{1}{n}, n\right]$

Note that the index set does <u>not</u> need to be a set of integers!

Of course, it often is. And so when it isn't, we'll use $\alpha$ or $x$ for the index ex.

**10.** **(a)** $\bigcup_{x\in[0,1]} [x,1]\times[0,x^2] =$ **(b)** $\bigcap_{x\in[0,1]} [x,1]\times[0,x^2] =$

Let $A_x = [x,1] \times [0,x^2]$