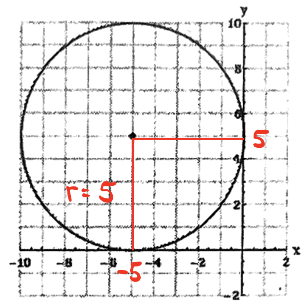
195, PreCalculus | Final Exam | No Electronics

Name *ANSWER KEY*



1. (4 points) Find an equation of the circle shown.

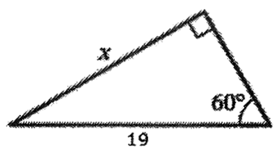
IN GENERAL, CENTER (h,k) & RADIUS r

$\longrightarrow (x-h)^2+(y-k)^2=r^2$

HERE, CENTER $(-5,5)$ & RADIUS 5

1. $(x+5)^2+(y-5)^2=25$

2. (4 points) Find the side



labeled x. You may leave sin, cos, or tan in your answer.

$\sin\theta=\frac{\text{OPP}}{\text{HYP}} \longrightarrow \sin 60^\circ=\frac{x}{19}$

$\longrightarrow x=19\sin 60^\circ=19\cdot\frac{\sqrt{3}}{2}$

2. $\frac{19\sqrt{3}}{2}$ or $19\sin 60^\circ$

3. (4 points) Solve the inequality $x^2+11x\le -30$. Write your answer in interval notation.

$x^2+11x+30\le 0$

$x^2+11x+30=0$

$(x+6)(x+5)=0$

$x=-6,-5$

SPLIT # LINE INTO INTERVALS

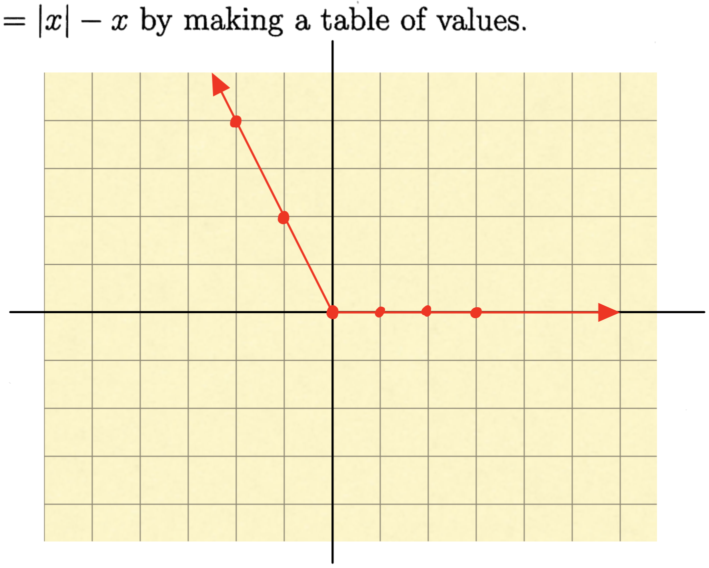
SIGN TABLE

		-6		-5	
$(x+6)(x+5)$:	$+$		$-$		$+$
$(x+6)$	$-$		$+$		$+$
$(x+5)$	$-$		$-$		$+$

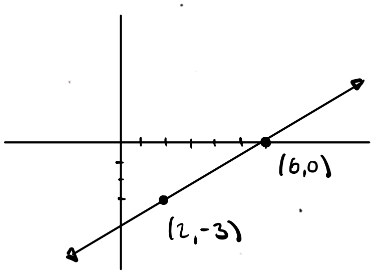
3. $[-6,5]$

4. (4 points) Sketch the graph of $y=|x|-x$ by making a table of values.

x	$y=\lvert x\rvert-x$
-3	6
-2	4
-1	2
0	0
1	0
2	0
3	0



5. (4 points) Find an equation of the line with x-intercept 6 and passing through the point $(2, -3)$.



Slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-3)}{6-2} = \frac{3}{4}$

Point-Slope EQ: $y - y_1 = m(x - x_1)$

Use $x_1 = 6,\ y_1 = 0$, or $x_1 = 2,\ y_1 = -3$.

$y + 3 = \frac{3}{4}(x-2)$, or

$y = \frac{3}{4}(x-6)$, or

5. $y = \frac{3}{4}x - \frac{9}{2}$

6. (4 points) Let $f(x) = 9 - x^2$. Find and simplify the difference quotient $\frac{f(1+h)-f(1)}{h}$ completely.

$f(1+h) = 9 - (1+h)^2 = 9 - (1 + 2h + h^2) = 8 - 2h - h^2$

$f(1) = 9 - 1^2 = 8$

$\frac{f(1+h) - f(1)}{h} = \frac{8 - 2h - h^2 - 8}{h} = \frac{h(-2-h)}{h}$

6. $-2 - h$

7. (4 points) Solve $\log(x) + \log(x-3) = 1$ for x.

$\log(x^2 - 3x) = 1$

$10^{\log(x^2-3x)} = 10^1$

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

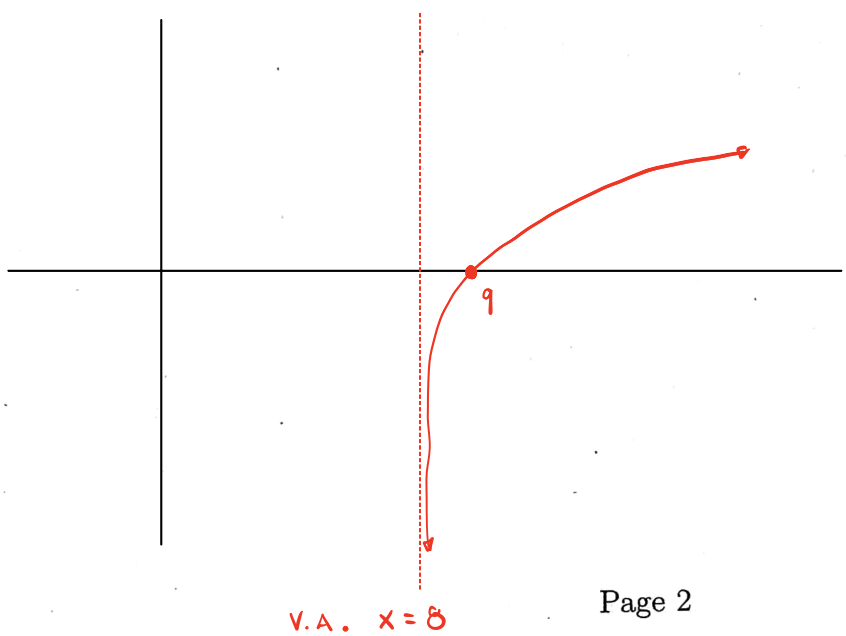
$(x-5)(x+2) = 0$

$x = 5$ ✓

$x = -2$ $\log(-2-3)$ undefined

7. 5

8. (4 points) Sketch the graph of $y = \log_8(x - 8)$. Find all the intercepts and asymptotes and label them clearly on your graph.



9. (4 points) Solve $5^{3x-4} = \frac{1}{25}$ for x.

$5^{3x-4} = 5^{-2}$

$\log_5(5^{3x-4}) = \log_5(5^{-2})$

$3x - 4 = -2$

$3x = 2$

$x = \frac{2}{3}$

9. $\frac{2}{3}$

10. (4 points) Find the inverse function of $f(x) = \frac{1}{x+3}$.

① SET $y = \frac{1}{x+3}$

② SOLVE FOR X

$y(x+3) = 1$

$x + 3 = \frac{1}{y}$

$x = \frac{1}{y} - 3$

③ SWITCH x & y

10. $f^{-1}(x) = \frac{1}{x} - 3$ $\left(= \frac{1-3x}{x} \right)$

11. (4 points) A table of values for $f(x)$ is given

x	1	2	3	4	5	6
f(x)	3	4	8	1	5	0

Determine the average rate of change of $f(x)$ between $x = 1$ and $x = 5$.

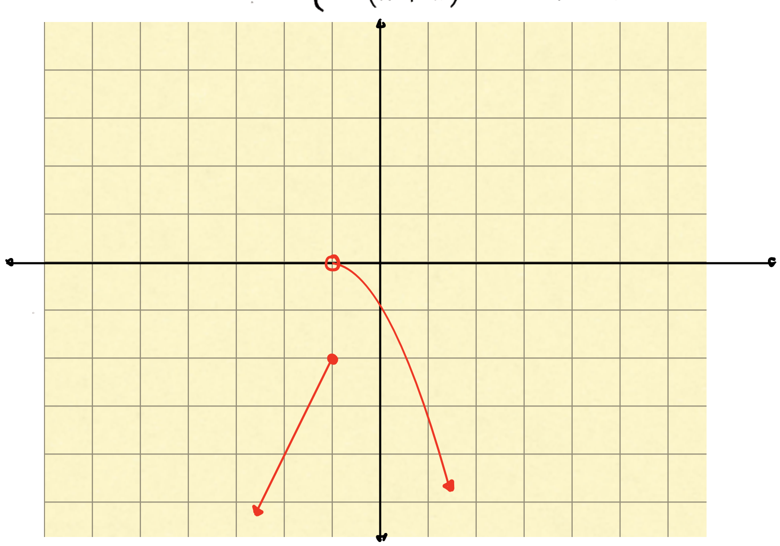
AVE RATE OF CHANGE OF f FROM a TO b IS $\frac{f(b) - f(a)}{b - a}$.

HERE WE HAVE $\frac{f(5) - f(1)}{5 - 1} = \frac{5 - 3}{4} = \frac{2}{4}$

11. $\frac{1}{2}$

12. (4 points) Sketch the graph of the piecewise defined function

$$f(x) = \begin{cases} 2x & \text{if } x \leq -1, \\ -(1+x)^2 & \text{if } x > -1. \end{cases}$$



13. (4 points) Simplify

$$\frac{(8x^3y^3)^{-\frac{1}{3}}}{(16x^4y^{-8})^{\frac{1}{2}}}$$

completely, writing your answer with only positive exponents.

$$= \frac{8^{-1/3}\, x^{-1}\, y^{-1}}{16^{1/2}\, x^{2}\, y^{-4}} = \frac{y^4}{2 \cdot 4\, x^2\, x\, y} = \frac{y^3}{8x^3}$$

13. $\frac{y^3}{8x^3}$

14. (4 points) A bacteria culture starts with 1000 bacteria. After 1 hour there are 2500 bacteria. Assuming the size of the culture grows exponentially, find the time required for there to be 5000 bacteria. (You may leave ln, log, and (or) e in your answer).

$P(t) = 1000e^{rt}$

$P(1) = 1000e^{r} = 2500$

$e^{r} = 2.5$

$(r = \ln(2.5))$

$P(t) = 1000(2.5)^t = 5000$

$2.5^t = 5$

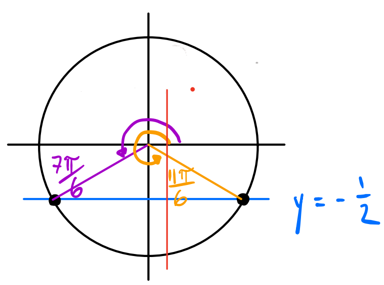
$t\ln(2.5) = \ln(5)$

$t = \frac{\ln(5)}{\ln(2.5)}$

14. $\frac{\ln(5)}{\ln(2.5)}$ or $\log_{2.5}(5)$

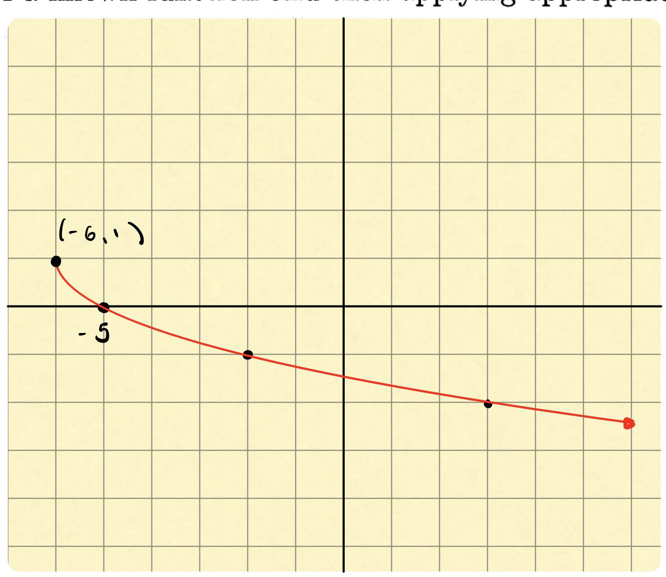
15. (4 points) Find all solutions t to $2\sin t + 1 = 0$ for $0 \le t \le 2\pi$.

$\sin t = -\frac{1}{2}$

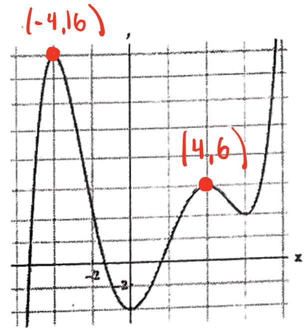


15. $\frac{7\pi}{6}, \frac{11\pi}{6}$

16. (4 points) Graph the function $y = 1 - \sqrt{x+6}$, not by plotting points, but by starting with the graph of a known function and then applying appropriate transformations.



17. (4 points) Find the the local maximum(s) of the graph.



LOCAL MAX VALUES: 16 (AT $x = -4$), & 6 (AT $x = 4$)

17. 16 (AT $x=-4$), & 6 (AT $x=4$)

18. (4 points) Perform the subtraction $\frac{5}{x(2x-3)} - \frac{6}{(2x-3)^2}$ and simplify as one fraction.

$$\frac{5(2x-3) - 6x}{x(2x-3)^2} = \frac{4x-15}{x(2x-3)^2}$$

18. $\frac{4x-15}{x(2x-3)^2}$

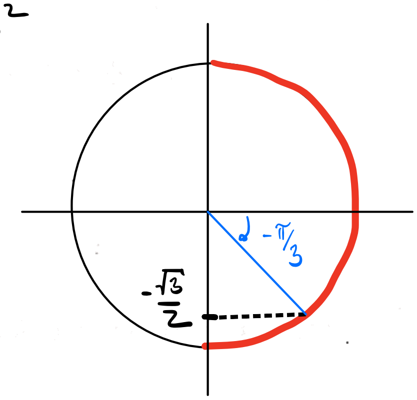
(or $\frac{4x-15}{4x^3-12x^2+9x}$)

19. (4 points) Evaluate $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = t$ IF

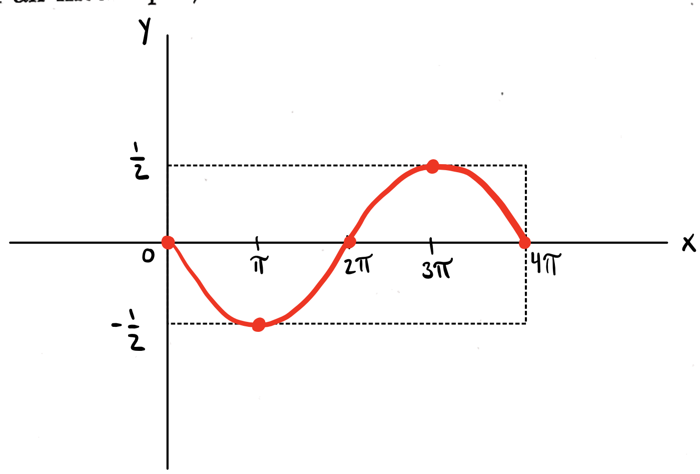
(1) $\sin t = -\frac{\sqrt{3}}{2}$, AND

(2) $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$.



19. $-\frac{\pi}{3}$

20. (4 points) Sketch the graph of one complete periods of the function $y = -\frac{1}{2}\sin\left(\frac{1}{2}x\right)$. Label all intercepts, maximums, and minimums.



AMPLITUDE : $\frac{1}{2}$

PERIOD : $\frac{2\pi}{1/2} = 4\pi$

21. (4 points) Evaluate $\sin\frac{\pi}{12}$.

$= \sin\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

21. $\frac{\sqrt{6}-\sqrt{2}}{4}$

22. (4 points) Find $\tan t$ if $\sin t = -\frac{4}{5}$ and $\cos t > 0$.

$\sin^2 t + \cos^2 t = 1 \Rightarrow \left(-\frac{4}{5}\right)^2 + \cos^2 t = 1$

$\Rightarrow \cos^2 t = 1 - \frac{16}{25} = \frac{9}{25} \Rightarrow \cos t = \oplus\sqrt{\frac{9}{25}} = \frac{3}{5}$

$\tan t = \frac{\sin t}{\cos t} = \frac{-4/5}{3/5} = -4/3$

22. $\frac{-4}{3}$

23. (4 points) Solve $\sin(2t) - \cos(t) = 0$ for t when $-\pi \leq t \leq \pi$.

$2\sin(t)\cos(t) - \cos(t) = 0$

$\cos(t)\,(2\sin(t) - 1) = 0$

$\cos(t) = 0 \Rightarrow t = \pm\frac{\pi}{2}$ $\quad 2\sin(t) - 1 = 0 \Rightarrow \sin t = \frac{1}{2} \Rightarrow t = \frac{\pi}{6}, \frac{5\pi}{6}$

23. $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

24. (4 points) Find all real solutions x to $\frac{6}{x} = \frac{8}{7x} + 1$.

MULT. BOTH SIDES BY LCD $7x$: $42 = 8 + 7x$

$34 = 7x$

$x = \frac{34}{7}$

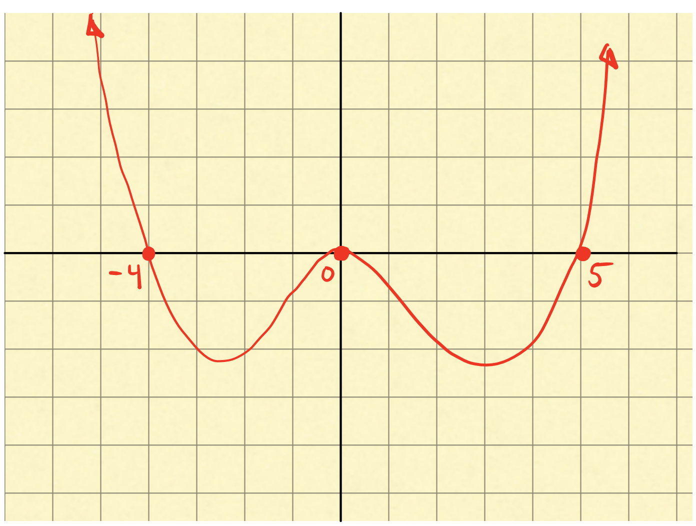
24. $\frac{34}{7}$

25. (4 points) Sketch the graph of the polynomial function $P(x) = x^4 - x^3 - 20x^2$. Label all intercepts.

$P(x) = x^2(x^2 - x - 20)$

$P(x) = x^2(x-5)(x+4)$

Root	-4	0	5
Multiplicity	1	2	1
Bounce/Cross	Cross	Bounce	Cross



END BEHAVIOR:

$P(x) \approx x^4$ ↑∪↑