Quiz 1

Name: *Answer Key* _____ Section: _____

> Answer all 6 questions for a total of 100 points. Write your solutions in the space provided and put a box around your final answers.

1. Solve for x.

   (a) (10 points) $3e^{x/2} + 1 = 13$

   $3e^{x/2} = 12$

   $\boxed{x = 2\ln 4}$ or $\ln 16$

   $e^{x/2} = 4$

   $\dfrac{x}{2} = \ln 4$

   (b) (10 points) $\ln(x+1) = \ln(x) + 1$
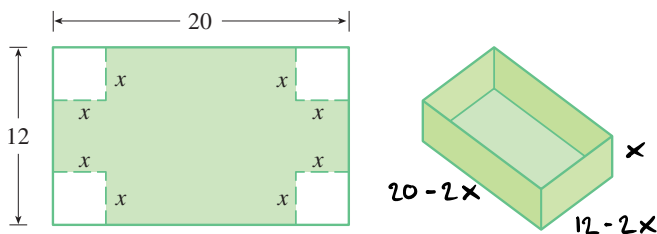
   $\ln(x+1) - \ln(x) = 1$ $x+1 = ex$

   $\ln\left(\dfrac{x+1}{x}\right) = 1$ $1 = ex - x = x(e-1)$

   $\boxed{\dfrac{1}{e-1} = x}$

   $\dfrac{x+1}{x} = e$

2. (16 points) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 inches by 20 inches by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.

20 - 2x

12 - 2x

$\boxed{V = x(20-2x)(12-2x)}$

OR ANYTHING
EQUIVALENT

$= 4x(10-x)(6-x) = 4x(x^2 - 16x + 60)$

$= 4x^3 - 64x^2 + 240x$

3. Let

$$f(x) = \frac{x-1}{x^2+1}, \quad g(x) = 3x+2, \quad \text{and} \quad h(x) = \frac{1}{\sqrt{x}}.$$

(a) (8 points) Find $f(g(0))$, $g(h(9))$, and $h(f(5))$.

$$f(g(0)) = f(3(0)+2) = f(2) = \frac{2-1}{2^2+1} = \boxed{\frac{1}{5}}$$

$$g(h(9)) = g\left(\frac{1}{\sqrt{9}}\right) = g\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)+2 = \boxed{3}$$

$$h(f(5)) = h\left(\frac{5-1}{5^2+1}\right) = h\left(\frac{4}{26}\right) = \frac{1}{\sqrt{\frac{4}{26}}} = \boxed{\frac{\sqrt{26}}{2}}$$

(b) (10 points) Find $h(g(x))$ and state its domain using interval notation.

$$h(g(x)) = h(3x+2) = \boxed{\frac{1}{\sqrt{3x+2}}}$$

DOMAIN: CANNOT TAKE $\sqrt{}$ OF NEG #'s :   CANNOT DIVIDE BY 0 :

$$3x+2 \geq 0 \qquad\qquad \sqrt{3x+2} \neq 0$$
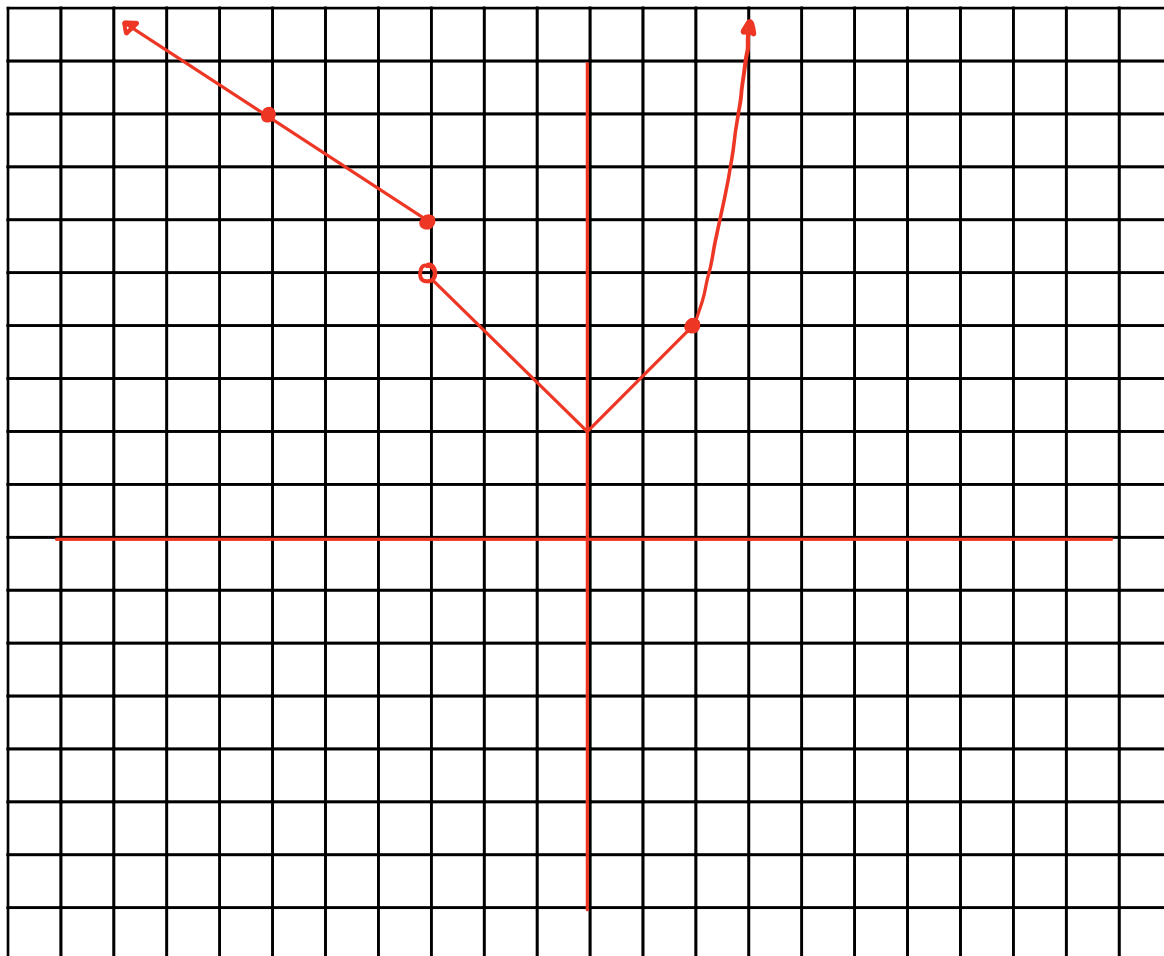$$3x \geq -2 \qquad\qquad\qquad x \neq -\frac{2}{3}$$
$$x \geq -\frac{2}{3}$$

$$\boxed{\left(-\frac{2}{3}, \infty\right)}$$

(c) (6 points) Find the averge rate of change of $f(x)$ over the interval $[2,4]$.

$$\frac{f(4)-f(2)}{4-2} = \frac{\frac{3}{17} - \frac{1}{5}}{4-2} = \frac{1}{2}\left(\frac{(5)(3)-(17)(1)}{(5)(17)}\right)$$

$$= \frac{1}{2}\left(\frac{-2}{85}\right) = \boxed{-\frac{1}{85}}$$

4. (16 points) Sketch the graph of $f(x) = \begin{cases} 8 - \dfrac{2x}{3} & \text{if } x \le -3 \\ |x| + 2 & \text{if } -3 < x < 2 \\ x^2 & \text{if } x \ge 2 \end{cases}$.

5. (8 points) Suppose the graph

$$y = 2e^x - x^2 + \frac{1}{x}$$

is shifted 4 to the left and 3 units up. Give an equation for the new, transformed graph.

$$y = 2e^{x+4} - (x+4)^2 + \frac{1}{x+4} + 3$$

6. Suppose $f(5) = 10$ and $f(10) = 25$.

    (a) (6 points) Find $f(x)$ assuming f is a linear function, i.e. $f(x) = mx + b$.

$$\text{SLOPE} \quad m = \frac{25 - 10}{10 - 5} = \frac{15}{5} = 3$$

$$\text{POINT-SLOPE EQ:} \quad y - 10 = 3(x - 4) \quad \text{or} \quad y = 3x - 2$$

$$\therefore \quad \boxed{f(x) = 3x - 2}$$

    (b) (10 points) Find $f(x)$ assuming f is an exponential function, i.e. $f(x) = ba^x$.

$$25 = ba^{10} \qquad \Rightarrow \qquad \frac{25}{10} = \frac{ba^{10}}{ba^5} = a^5 \qquad \Rightarrow \qquad a = 2.5^{1/5}$$
$$10 = ba^5$$

$$\text{THEN} \quad 10 = b\left(2.5^{1/5}\right)^5 = 2.5\,b \qquad \Rightarrow \qquad b = 4$$

$$\therefore \quad f(x) = 4\left(2.5^{1/5}\right)^x \quad \text{or} \quad \boxed{f(x) = 4(2.5)^{x/5}}$$