ex. (a) $f(x) = \sqrt{x+2}$

EXPRESSIONS INSIDE $\sqrt{\ }$ MUST BE ≥ 0.

SET $x+2 \geq 0$

$x \geq -2 \longrightarrow \boxed{[-2, \infty)}$

(b) $g(x) = \dfrac{1}{x^2 - 5x}$

THE DENOMINATOR CANNOT $= 0$.

SET $x^2 - 5x \neq 0$

$x(x-5) \neq 0$

$x \neq 0$ AND $x - 5 \neq 0 \longrightarrow \boxed{(-\infty, 0) \cup (0, 5) \cup (5, \infty)}$

$x \neq 5$

(c) $h(x) = \dfrac{\sqrt{x+2}}{x^2 - 5x}$

COMBINE $x \geq -2$, $x \neq 0$, $x \neq 5$

$\boxed{[-2, 0) \cup (0, 5) \cup (5, \infty)}$

ex. $f(x) = x^2 - \dfrac{1}{x}$ $\qquad g(x) = 8 - 5x$

(a) FIND $f \circ g(x)$

$f(g(x)) = f(8-5x) = \boxed{(8-5x)^2 - \dfrac{1}{8-5x}}$

(b) FIND $g \circ f(x)$

$g(f(x)) = g\left(x^2 - \dfrac{1}{x}\right) = \boxed{8 - 5\left(x^2 - \dfrac{1}{x}\right)}$

(c) FIND $g \circ g(x)$

$g(g(x)) = g(8-5x) = \boxed{8 - 5(8-5x)}$

ex. Suppose f is exponential, $f(x) = ca^x$. If $f(20) = 24$ and $f(30) = 144$, find $f(x)$.

$$f(x) = ca^x \qquad f(20) = \boxed{ca^{20} = 24} \quad (1)$$
$$f(20) = 24$$
$$f(30) = 144 \qquad f(30) = \boxed{ca^{30} = 144} \quad (2)$$

$$\frac{(2)}{(1)} : \quad \frac{ca^{30}}{ca^{20}} = \frac{144}{24} = 6 \quad \Rightarrow \quad a^{10} = 6$$
$$a = 6^{1/10}$$

Now (1): $c\left(6^{1/10}\right)^{20} = 24$

$$36c = 24 \quad \Rightarrow \quad c = \frac{24}{36} = \frac{2}{3}$$

$$\therefore \boxed{f(x) = \frac{2}{3}\left(6^{1/10}\right)^x \quad \text{or} \quad f(x) = \frac{2}{3} \cdot 6^{x/10}}$$

ex. (a) What is $\log_3\left(\frac{1}{9}\right)$?

Set $\log_3\left(\frac{1}{9}\right) = x \longrightarrow 3^x = \frac{1}{9}$

Since $3^2 = 9$, we have $3^{-2} = \frac{1}{9} \Rightarrow x = \boxed{-2}$

(b) Solve: $\log_3(x^2 + 6x) = 3$

Equivalent to $3^3 = x^2 + 6x$
$$0 = x^2 + 6x - 27 = (x+9)(x-3)$$

$$\boxed{x = -9, 3}$$

3. Revenue Let $R(t)$ be the monthly revenue (in thousands of dollars) of a restaurant t months after the restaurant opened. If $R(6) = 154.2$ and $R(9) = 179.7$, compute the average rate of change for $6 \leqslant t \leqslant 9$. What does your result mean in this context?

RESULT MEAN IN THIS CONTEXT?

$$\frac{R(9) - R(6)}{9 - 6} = \frac{179.7 - 154.2}{9 - 6} = \frac{25.5}{3} = \boxed{8.5}$$

Over the three month period 6-9 months after opening, the restaurant's monthly revenue increased by $8,500 per month, on average.

The graph of f is given.

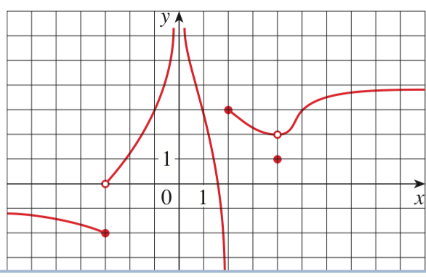
(a) Find each limit, or explain why it does not exist.

(i) $\lim_{x \to 2^+} f(x)$ **(ii)** $\lim_{x \to -3^+} f(x)$

(iii) $\lim_{x \to -3} f(x)$ **(iv)** $\lim_{x \to 4} f(x)$

(v) $\lim_{x \to 0} f(x)$

(b) At what numbers is f not continuous? Explain.



(a) (i) 3

(ii) 0

(iii) DNE : $\lim_{x \to 3^-} f(x) = -2$, $\lim_{x \to 3^+} f(x) = 0$ NOT EQUAL

(iv) 2

(v) DNE BECAUSE $\lim_{x \to 0} f(x) = \infty$.

(b) -2 : $\lim_{x \to -2} f(x)$ D.N.E.

0 : $\lim_{x \to 0} f(x)$ D.N.E. AND $f(0)$ UNDEFINED

2 : $\lim_{x \to 2} f(x)$ D.N.E.

4 : $\lim_{x \to 4} f(x) \neq f(4)$

$2 \neq 1$

7–14 ■ Evaluate the limit.

7. $\lim_{x \to 1} (5x^2 - 4x + 5)$

8. $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

9. $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

10. $\lim_{t \to 2} \dfrac{t^2 - 4}{t^2 + 3t - 10}$

11. $\lim_{t \to 0} 4e^{-2t}$

12. $\lim_{b \to 1} (\ln b)^2$

8 PLUG IN $x = 3$: $\dfrac{3^2 - 9}{3^2 + 2(3) - 3} = \dfrac{0}{12} = \boxed{0}$

9 $\lim_{x \to -3} \dfrac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-1)}$

$x = -3$: $\dfrac{-3-3}{-3-1} = \dfrac{-6}{-4} = \boxed{\dfrac{3}{2}}$

10 $\lim\limits_{t \to 2} \dfrac{(t+2)\cancel{(t-2)}}{(t+5)\cancel{(t-2)}}$, $t=2$: $\dfrac{2+2}{2+5} = \boxed{\dfrac{4}{7}}$

11 PLUG IN $t=0$: $4e^{-2(0)} = 4e^{0} = 4 \cdot 1 = \boxed{4}$

12 PLUG IN $b=1$: $(\ln 1)^2 = 0^2 = \boxed{0}$

ex. SETUP A LIMIT FOR $f'(3)$ WHEN $f(x) = \ln(x+1)$. DO NOT EVALUATE.

$$\boxed{\begin{array}{l} f'(3) = \lim\limits_{x \to 3} \dfrac{\ln(x+1) - \ln(3+1)}{x-3}, \text{ OR} \\ f'(3) = \lim\limits_{h \to 0} \dfrac{\ln(3+h+1) - \ln(3+1)}{h} \end{array}}$$

ex. GIVE EQUATION OF TANGENT LINE TO $y = \dfrac{3}{x+4}$ AT $(-5, -3)$

$a = -5$ $f(a) = -3$

$f(x) = \dfrac{3}{x+4}$

$y = f(a) + f'(a)(x-a)$

$y = -3 + f'(-5)(x-(-5))$

$f'(-5) = \lim\limits_{x \to -5} \dfrac{f(x) - f(-5)}{x-(-5)} = \lim\limits_{x \to -5} \dfrac{\frac{3}{x+4} + 3}{x+5}$

$= \lim\limits_{x \to -5} \dfrac{1}{x+5}\left(\dfrac{3}{x+4} + \dfrac{3(x+4)}{x+4}\right)$

$= \lim\limits_{x \to -5} \dfrac{3+3x+12}{(x+5)(x+4)} = \lim\limits_{x \to -5} \dfrac{15+3x}{(x+5)(x+4)} = \lim\limits_{x \to -5} \dfrac{3\cancel{(x+5)}}{\cancel{(x+5)}(x+4)}$

$x = -5$: $\dfrac{3}{-5+4} = \dfrac{3}{-1} = -3$

$$\boxed{y = -3 - 3(x+5) \quad \text{OR} \quad y = -3x - 18}$$

ex. FIND $f'(x)$ WHEN $f(x) = \sqrt{2x+1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)+1}^{\,2} - \sqrt{2x+1}^{\,2}}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)} \longrightarrow \begin{array}{l} 2(x+h)+1 - (2x+1) \\ \cancel{2x} + 2h + \cancel{1} - \cancel{2x} - \cancel{1} \end{array}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$$

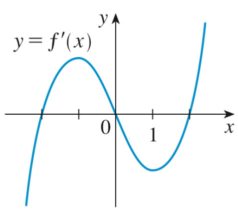
$$h = 0: \quad \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}} = \boxed{\frac{1}{\sqrt{2x+1}}}$$

45. The graph of the *derivative* f' of a function f is given.

(a) On what intervals is f increasing or decreasing?

(b) At what values of x does f have a local maximum or minimum?

(c) Where is f concave upward or downward?



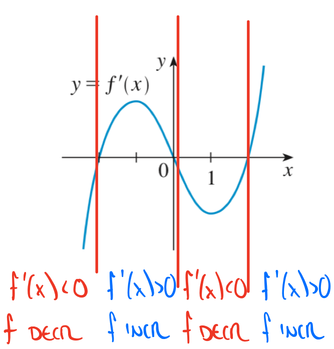
a. INCREASING $(-2,0) \cup (2,\infty)$
DECREASING $(-\infty,-2) \cup (0,2)$

b. LOCAL MAX @ $x = 0$
LOCAL MIN @ $x = -2$, $x = 2$

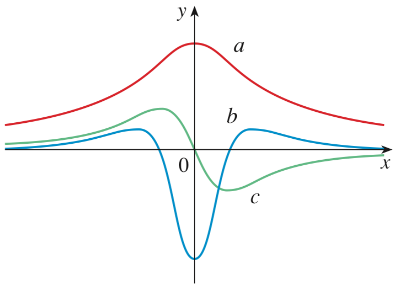
c. CONC. UP : $(-\infty,-1) \cup (1,\infty)$
CONC. DOWN : $(-1,1)$

WHEN $f'(x)$ IN NEG., $f(x)$ IS DECREASING
WHEN $f'(x)$ IN POS., $f(x)$ IS INCREASING

WHEN $f'(x)$ IS INCREASING, $f(x)$ IS CONCAVE UP
WHEN $f'(x)$ IS DECREASING, $f(x)$ IS CONCAVE DOWN

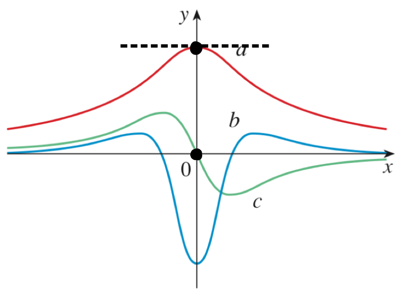


46. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



$f(x) = a$: b & c HAVE HORIZONTAL TANGENT LINES (WITH SLOPE 0) AT MULTIPLE POINTS, BUT GRAPH a IS NEVER 0. SO IT CANNOT BE THE GRAPH OF THE DERIVATIVE OF EITHER

$f'(x) = c$: GRAPH a HAS ONE POINT WHERE TANGENT LINE HAS SLOPE 0 (WHEN $x = 0$) & GRAPH b IS 0 ONCE (WHEN $x = 0$).



$f''(x) = b$: GRAPH c HAS 2 POINTS WHERE TANGENT LINES HAVE SLOPE 0 AND FOR THESE SAME X-VALUES, GRAPH b IS 0.