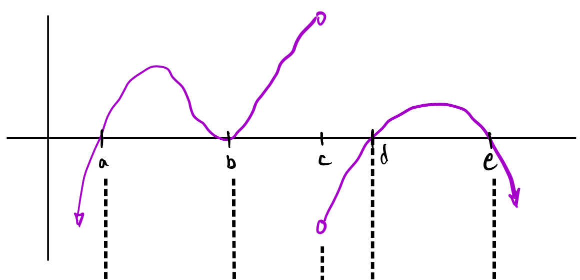
§4.3 DERIVATIVES & THE SHAPES OF CURVES

INCREASING/DECREASING

RECALL FROM §2.4 THAT A FUNCTION f HAS

- IF $f'(x) > 0$ ON AN INTERVAL THEN f IS INCREASING ON THAT INTERVAL
- IF $f'(x) < 0$ ON AN INTERVAL THEN f IS DECREASING ON THAT INTERVAL

NOTE: $f'(x)$ CAN ONLY CHANGE SIGN AT CRITICAL NUMBERS



$y = f'(x)$

a, b, c, d, e ARE ALL CRITICAL NUMBERS.

INTERVAL	$(-\infty, a)$	(a, b)	(b, c)	(c, d)	(d, e)	(e, ∞)
$f'(x)$	$-$	$+$	$+$	$-$	$+$	$-$
$f(x)$	DECR.	INCR.	INCR.	DECR.	INCR.	DECR.

← CLOSED INTERVALS WOULD ALSO BE OK.

■ The First Derivative Test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.

ex. FIND INTERVALS ON WHICH $f(x) = 4x^3 - 11x^2 - 20x + 7$ IS INCREASING & DECREASING. FIND ALL LOCAL MAX/MIN VALUES.

1. FIND CRITICAL NUMBERS
2. DETERMINE SIGN OF $f'(x)$ ON EACH INTERVAL BETWEEN CRITICAL NUMBERS (SIGN TABLE)
3. $f'(x)$ POS $\Rightarrow$ $f(x)$ INCREASING
   $f'(x)$ NEG $\Rightarrow$ $f(x)$ DECREASING

(n CRITICAL POINTS)

($n+1$ INTERVALS)

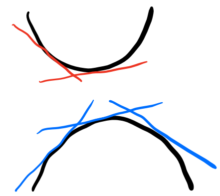
$f'(x) = 12x^2 - 22x - 20 = 2(6x^2 - 11x - 10) = 2(3x+2)(2x-5)$

CRITICAL NUMBERS $-\frac{2}{3}, \frac{5}{2}$. $f(-\frac{2}{3}) \approx 14.26$, $f(\frac{5}{2}) = -49.25$

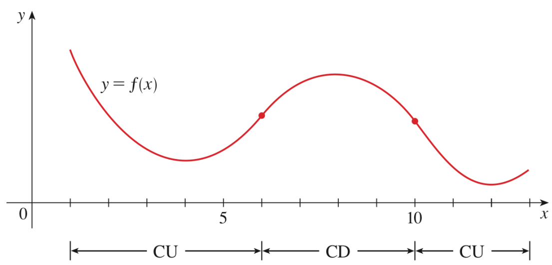
CONCAVITY

RECALL FROM §2.4 THAT A FUNCTION f IS

- CONCAVE UP ON AN INTERVAL IF $f''(x) > 0$ ON THAT INTERVAL
  & $f'(x)$ INCREASING, $y = f(x)$ ABOVE TANGENT LINES
- CONCAVE DOWN ON AN INTERVAL IF $f''(x) < 0$ ON THAT INTERVAL
  & $f'(x)$ DECREASING, $y = f(x)$ BELOW TANGENT LINES



Definition A point on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at that point.



ex. Let $f(x) = (x^2-1)^3$.

(a) FIND INTERVALS WHERE f IS INCREASING & DECREASING

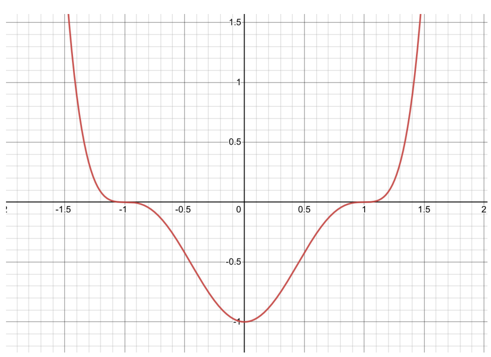
(b) FIND ALL LOCAL MAX/MIN VALUES

(c) FIND INTERVALS WHERE f IS CONCAVE UP / DOWN.

(d) FIND ALL INFLECTION POINTS.

$$f'(x) = 3(x^2-1)^2(2x) = 6x(x+1)^2(x-1)^2$$

$$f''(x) = 6(x^2-1)(2x)^2 + 6(x^2-1)^2 = 6(x^2-1)(4x^2+x^2-1) = 6(x+1)(x-1)(\sqrt{5}x+1)(\sqrt{5}x-1)$$



CREATE A SIGN TABLE FOR $f''(x)$.

■ **The Second Derivative Test** Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

ex. USE 2^{nd} DERIVATIVE TEST TO FIND ALL LOCAL MIN/MAX VALUES OF $f(x) = \dfrac{x^2}{x-1}$

$$f'(x) = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

CRITICAL NUMBERS: $x = 0, 2$

$$f''(x) = \frac{(2x-2)(x-1)^2 - (x^2-2x)2(x-1)}{(x-1)^4}$$

$f''(0) = -2 \Rightarrow f(0) = 0$ IS LOCAL MAX ✓

$f''(2) = 2 \Rightarrow f(2) = 4$ IS LOCAL MIN ✓