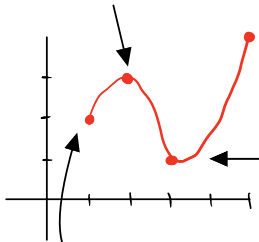
§4.2 Maximum & Minimum Values

Def: $f(c)$ is an absolute maximum value of f if $f(c) \geq f(x)$
absolute minimum value $f(c) \leq f(x)$
for all x in the domain of f.

$f(c)$ is a local maximum value of f if $f(c) \geq f(x)$
local minimum value $f(c) \leq f(x)$
for all x near c, that is for all x in an open interval that contains c.

Local max value 3
at $x = 2$

Absolute max value 4
at $x = 5$

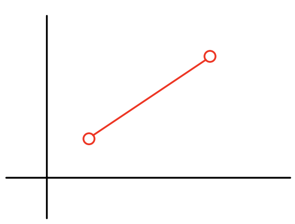


Local min value &
absolute min value 1
at $x = 3$

Neither local/absolute
minimum

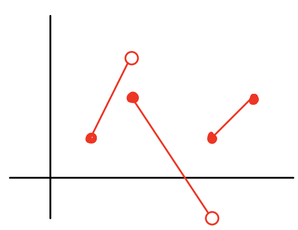
Note: Local min/max cannot occur at an endpoint of the domain of f.
Absolute max/min can.

(3) ■ The Extreme Value Theorem If f is continuous on a closed interval, then it always attains an absolute maximum value and an absolute minimum value on that interval.



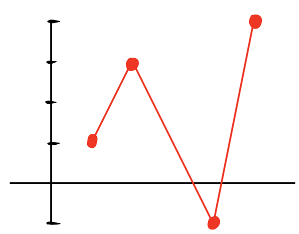
No abs. max.
No abs. min.

f not defined on closed interval



No abs. max.
No abs. min.

f not continuous

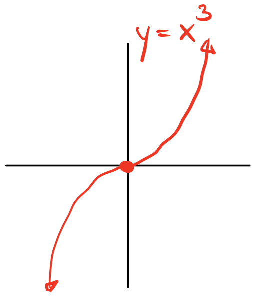


Abs max value 4
Abs min value -1

✓

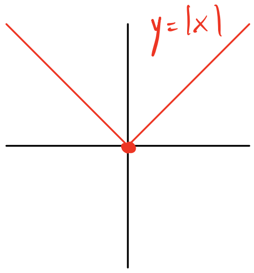
(4) ■ Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

WHY? OTHERWISE, IF $f'(c) \neq 0$ THEN THE GRAPH $y = f(x)$ HAS TANGENT LINE AT $(c, f(c))$ THAT IS NOT HORIZONTAL. THUS, FOLLOWING THE CURVE A SMALL DISTANCE IN ONE DIRECTION WILL LEAD TO HIGHER POINTS (SO $f(c)$ CANNOT BE A LOCAL MAX VALUE), & IN THE OTHER DIRECTION TO LOWER POINTS (SO $f(c)$ CANNOT BE A LOCAL MIN VALUE).



WARNING: IT IS <u>NOT</u> TRUE THAT IF $f'(c) = 0$ THEN $f(c)$ IS A LOCAL MAX/MIN VALUE.

<u>e.g.</u> IF $f(x) = x^3$ THEN $f'(0) = 0$ BUT $f(0) = 0$ IS <u>NOT</u> A LOCAL MAX/MIN VALUE OF f.



IT IS <u>NOT</u> TRUE THAT IF $f(c)$ IS A LOCAL MAX/MIN THEN $f'(c) = 0$

<u>e.g.</u> IF $f(x) = |x|$ THEN $f(0) = 0$ IS A LOCAL MIN BUT $f'(0) \neq 0$.

(5) ■ Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

FERMAT'S THM RESTATED:

(6) ■ If f has a local maximum or minimum at c, then c is a critical number of f.

<u>**ex.**</u> FIND THE CRITICAL POINTS OF

(a) $f(x) = \sqrt{1 - x^2}$

(b) $g(x) = x^{4/5}(x-4)^2$

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the critical numbers of f in the interval (a, b) and compute the values of f at these numbers.
2. Find the values of f at the endpoints of the interval.
3. The largest of the output values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

ex. FIND THE ABSOLUTE MAX & MIN VALUES OF f ON THE GIVEN INTERVAL.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 1$; $[-2, 3]$

(b) $g(x) = e^{-x} - e^{-2x}$; $[0, 1]$

(c) $h(x) = x\sqrt{4 - x^2}$; $[-1, 2]$