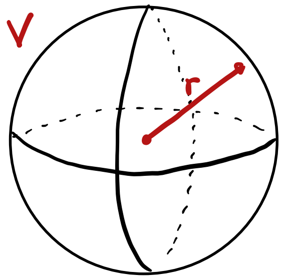
CH. 4 APPLICATIONS OF DIFFERENTIATION

§4.1 RELATED RATES

APPLICATION OF IMPLICIT DIFFERENTIATION TO RELATED QUANTITIES THAT CHANGE OVER TIME.

MAIN IDEA: THESE QUANTITIES ARE FUNCTIONS OF t.

ex. A SPHERICAL BALLOON IS BEING INFLATED SUCH THAT ITS VOLUME IS INCREASING AT A RATE OF $\frac{1}{4}\ m^3/s$.
WHEN THE RADIUS OF THE BALLOON IS 1 m,
FIND THE RATE AT WHICH THE RADIUS IS INCREASING.



(1) PICTURE, NOTATION — USE VARIABLES FOR ALL QUANTITIES THAT CHANGE OVER TIME

(2) IDENTIFY GIVEN, UNKNOWN RATES

GIVEN $\frac{dV}{dt} = \frac{1}{4}\ m^3/s$ FIND $\frac{dr}{dt}$

(3) WRITE EQUATIONS RELATING THE QUANTITIES

$$V = \frac{4}{3}\pi r^3$$

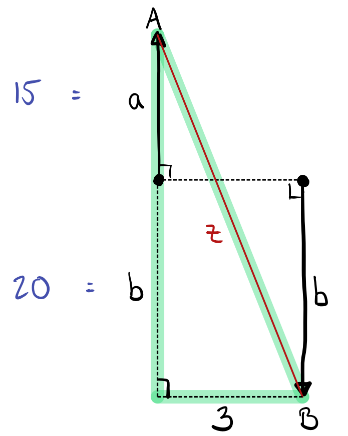
(4) TAKE $\frac{d}{dt}$ OF BOTH SIDES → EQ RELATING THE RATES (i.e. DERIVATIVES)

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

(5) EVALUATE BY PLUGGING IN ALL KNOWN QUANTITIES.

$$\left.\frac{dr}{dt}\right|_{r=1} = \frac{\left(\frac{1}{4}\ m^3/s\right)}{4\pi\,(1\ m)^2} = \boxed{\frac{1}{16\pi}\ m/s}$$

ex. 2 SHIPS A & B BEGIN WITH B, 3 MILES EAST OF A.
SHIP A SAILS NORTH AT 15 mph & SHIP B SAILS SOUTH AT 20 mph.
AFTER 1 HR, AT WHAT RATE IS THE DISTANCE BETWEEN THE SHIPS INCREASING?



GIVEN : $\frac{da}{dt} = 15$ mph $\qquad \frac{db}{dt} = 20$ mph

FIND : $\frac{dz}{dt}$

RELATED QUANTITIES : $(a+b)^2 + 3^2 = z^2$

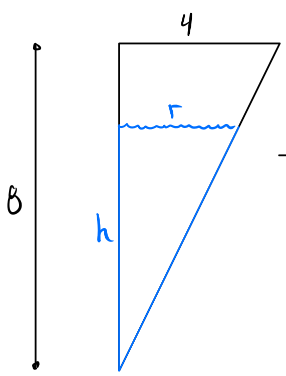
RELATED RATES : $\frac{d}{dt}\left[(a+b)^2 + 3^2\right] = \frac{d}{dt}\left[z^2\right]$

$$\cancel{2}(a+b)\left(\frac{da}{dt} + \frac{db}{dt}\right) + 0 = \cancel{2}z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{(a+b)\left(\frac{da}{dt} + \frac{db}{dt}\right)}{z}$$

$$\left.\frac{dz}{dt}\right|_{\substack{a=15 \\ b=20}} = \frac{(15+20)(15+20)}{\sqrt{(15+20)^2 + 3^2}} \approx 34.87 \text{ mph}$$

ex. WATER IS LEAKING OUT OF A TANK IN SHAPE OF INVERTED CONE WITH HEIGHT 8 m AND TOP RADIUS 4 m. IF WATER LEVEL IS DECREASING AT A RATE OF 2 cm PER MINUTE WHEN WATER LEVEL IS 6 m, FIND THE RATE AT WHICH WATER IS LEAKING FROM THE TANK.



$V = \frac{1}{3}\pi r^2 h$ $\qquad$ NOTE: WE CAN REMOVE r!

$$\frac{r}{h} = \frac{4}{8} \Rightarrow r = \frac{1}{2}h$$

Now $V = \frac{\pi}{12}h^3$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{\substack{h=6 \\ \frac{dh}{dt}=.02}} = \frac{\pi(6)^2}{4}(.02) = .18\pi \text{ m}^3/\text{MIN} \approx .57 \text{ m}^3/\text{MIN}.$$