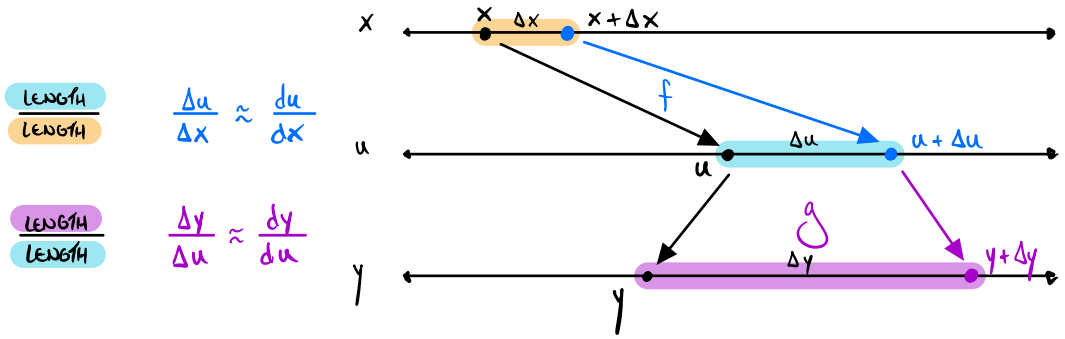
§3.4 THE CHAIN RULE

LET $y = f(u)$ AND $u = g(x)$

LET Δx BE VERY SMALL.



$$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x} \approx \frac{dy}{du} \cdot \frac{du}{dx}$$

CHAIN RULE: LET $y = f(u)$ AND $u = g(x)$.

THEN $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$.

ALTERNATELY, LET $y = f(u) = f(g(x))$ & $u = g(x)$

$\dfrac{dy}{du} = f'(u) = f'(g(x))$ $\qquad$ $\dfrac{du}{dx} = g'(x)$

THEN $\dfrac{d}{dx}\left[f(g(x)) \right] = f'(g(x))\, g'(x)$

ex. LET $y = e^{x^2}$. FIND $\dfrac{dy}{dx}$.

ex. LET $y = (x^2 + 3x - 1)^5$. FIND $\dfrac{dy}{dx}$.

(4) ■ The Power Rule Combined with the Chain Rule If n is any real number and $u = g(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

Alternatively, $$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

ex. Let $y = \sqrt{e^x + 2x}$. Find $\frac{dy}{dx}$.

ex. Let $y = \frac{3}{(5x^2+1)^4}$. Find $\frac{dy}{dx}$.

ex. Let $y = e^{\sqrt{x}}$. Find $\frac{dy}{dx}$.

ex. Let $y = 2^x$. Find $\frac{dy}{dx}$.

■ If $u = g(x)$ is a differentiable function, then

(5) $$\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$$

or, equivalently,

(6) $$\frac{d}{dx}e^{g(x)} = e^{g(x)} \cdot g'(x)$$

Note: $\frac{d}{dx}\left[a^x\right] = \frac{d}{dx}\left[e^{\ln(a)x}\right] = e^{\ln(a)x}\frac{d}{dx}\left[\ln(a)x\right] = e^{\ln(a)x} \cdot \ln(a) = a^x \ln a$

$$\boxed{\frac{d}{dx}\left[a^x\right] = a^x \ln(a)}$$

ex. Find $\frac{d}{dx}\left[\frac{3x+1}{(4x+3)^4}\right]$ (do not have to use Quotient Rule)