§2.3 RATES OF CHANGE AND DERIVATIVES

ex. RECALL GALILEO: DISTANCE ft FALLEN t SECONDS AFTER DROP

$s(t) = 4.9t^2$

AVERAGE RATE OF CHANGE OF s OVER $[5, 5+h]$ (CHANGE IN TIME) $= \dfrac{s(5+h)-s(5)}{5+h-5} = \dfrac{4.9(5+h)^2 - 4.9(5)^2}{h}$

$\dfrac{\Delta s}{\Delta t}$ = SPEED

$= \dfrac{4.9(25+10h+h^2-25)}{h} = \dfrac{4.9h(10+h)}{h}$

$= 49 + 4.9h \quad ft/s$

INSTANTANEOUS RATE OF CHANGE OF s AT $t=5$: $\lim_{h\to 0} 49 + 4.9h = 49 \quad ft/s.$

Def: THE INSTANTANEOUS RATE OF CHANGE OF $f(x)$ AT $x=a$ IS

$$f'(a) = \lim_{x\to a} \frac{f(a+h)-f(a)}{h}$$

(PROVIDED THAT LIMIT EXISTS)

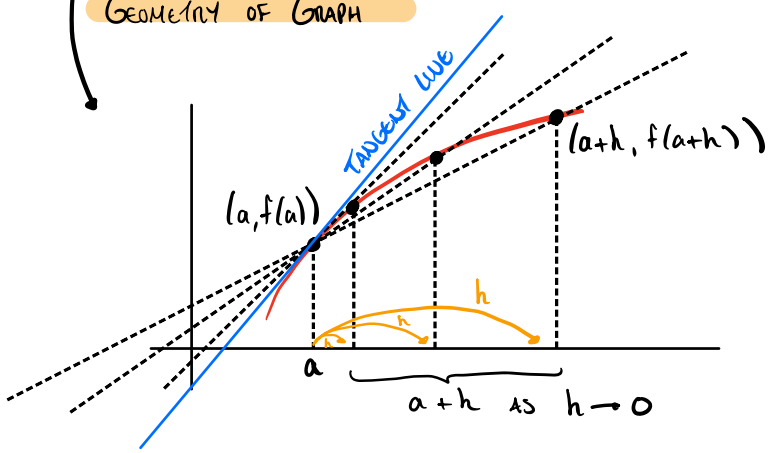
ALT

LET $x = a+h \quad (h = x-a)$

NOTE AS $h\to 0$, $x \to a$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

GEOMETRY OF GRAPH



SECANT LINES CONNECTING $(a\ f(a))$ & $(x, f(x))$ HAVE SLOPE

$$\frac{f(a+h)-f(a)}{h}$$

SLOPE OF TANGENT LINE TO $y=f(x)$ AT $(a, f(a))$ IS

$$m = f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

(PROVIDED THAT LIMIT EXISTS)

AN EQUATION OF THE TANGENT LINE TO $y=f(x)$ AT POINT $(a, f(a))$ IS

$y - f(a) = f'(a)(x-a) \longrightarrow y = f(a) + f'(a)(x-a)$

ex. (a) FIND INSTANTANEOUS RATE OF CHANGE OF $f(x) = 3x^2 - 2x + 5$ WHEN $x = 2$.

(b) FIND EQUATION OF TANGENT LINE TO GRAPH $y = 3x^2 - 2x + 5$ POINT ON GRAPH WITH $x = 2$.

ex. (a) FIND INSTANTANEOUS RATE OF CHANGE OF $f(x) = \frac{2}{x+1}$ WHEN $x = 3$.

(b) FIND EQUATION OF TANGENT LINE TO GRAPH $y = \frac{2}{x+1}$ AT POINT ON GRAPH WITH $x = 3$.

ex. (a) FIND INSTANTANEOUS RATE OF CHANGE OF $f(x) = \frac{2x+1}{3x+4}$ WHEN $x = 1$.

(USE ALT DEF. OF DERIVATIVE $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$)

(b) FIND EQUATION OF TANGENT LINE TO GRAPH $y = \frac{2x+1}{3x+4}$ AT POINT ON GRAPH WITH $x = 1$.

40. Bacteria population The number of bacteria after t hours in a controlled laboratory experiment is $n = f(t)$.

(a) What is the meaning of the derivative $f'(5)$? What are its units?

(b) Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger, $f'(5)$ or $f'(10)$? If the supply of nutrients is limited, would that affect your conclusion? Explain.

41. Fuel consumption The fuel consumption, measured in gallons per hour, of a car traveling at a speed of v miles per hour is $c = f(v)$.

(a) What is the meaning of the derivative $f'(v)$? What are its units?

(b) Write a sentence (in layman's terms) that explains the meaning of the equation $f'(20) = -0.05$.

43. Heating costs Let $H(t)$ be the daily cost, in dollars, to heat an office building when the outside temperature is t degrees Fahrenheit.

(a) What is the meaning of $H'(58)$? What are its units?

(b) Would you expect $H'(58)$ to be positive or negative? Explain.

46. Revenue function Let $R(p)$ be the commission a consignment shop takes when they sell an item at p dollars.

(a) Interpret the statement $R(450) = 110$.

(b) Interpret the statement $R'(450) = 0.25$.