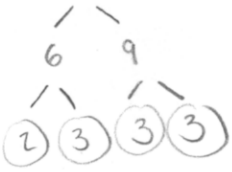
Name: *Answer Key* 1/24/2017

Math 71 Elementary Algebra Final Exam

Calculators are not allowed. Answers may be left as improper fractions, mixed numbers, or decimals. **Box your final answers**. If you need more space, you may continue your work on the back of the page.

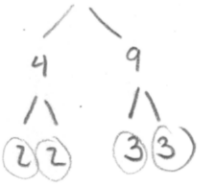
1. (6 points) Find the prime factorization of the following numbers.

(a) 54



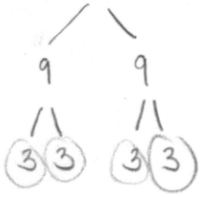
$54 = 2 \cdot 3^3$

(b) 36



$36 = 2^2 \cdot 3^2$

(c) 81



$81 = 3^4$

2. (12 points) Perform the following operations and simplify all answers.

(a) $\left(\frac{1}{36} - \frac{1}{54}\right) \cdot \frac{81}{5} \div \frac{21}{4}$

LCD = 108

$\frac{3-2}{108} \cdot \frac{81}{5} \cdot \frac{4}{21} = \frac{(1)(\cancel{3} \cdot 27)(4)}{(4 \cdot \cancel{27})(5)(\cancel{3} \cdot 7)} = \frac{1}{35}$

(b) $\left(108\left(\underbrace{\frac{1}{36}-\frac{1}{81}}_{\text{LCD}=324}\right)-1\right)^2+\frac{2}{3}$

$$\left(\cancel{108}\cdot\frac{9-4}{\cancel{324}_{3}}-1\right)^2+\frac{2}{3}=\left(\frac{5}{3}-1\right)^2+\frac{2}{3}$$

$$=\left(\frac{2}{3}\right)^2+\frac{2}{3}=\frac{4}{9}+\frac{6}{9}=\boxed{\frac{10}{9}}$$

(c) $\dfrac{\frac{1}{2}-\frac{1}{6}}{\frac{1}{3}+\frac{1}{4}}+\frac{3}{7}$

$$=\frac{\frac{1}{3}}{\frac{7}{12}}+\frac{3}{7}$$

$$=\frac{1}{\cancel{3}}\cdot\frac{\cancel{12}^{4}}{7}+\frac{3}{7}=\frac{4}{7}+\frac{3}{7}=\boxed{1}$$

3. (6 points) Evaluate each of the following.

(a) 7^{-2}

$$=\frac{1}{7^2}=\boxed{\frac{1}{49}}$$

(b) $\sqrt[3]{-64}$

$$=\boxed{-4}\qquad \text{BECAUSE}\quad (-4)^3=-64$$

(c) $8^{5/3}$

$$=\left(8^{1/3}\right)^5=\left(\sqrt[3]{8}\right)^5=2^5=\boxed{32}$$

4. (12 points) Let $x = 1\frac{1}{3}, y = 2\frac{2}{5}$, and $z = -\frac{3}{4}$. Evaluate the following expressions.

(a) $y(x - z^2)$

$$\frac{12}{5}\left(\frac{4}{3} - \left(-\frac{3}{4}\right)^2\right) = \frac{12}{5}\left(\frac{4}{3} - \frac{9}{16}\right) = \frac{12}{5}\left(\frac{64 - 27}{48}\right)$$

$$= \frac{\cancel{12}}{5} \cdot \frac{37}{\cancel{48}_{4}} = \boxed{\frac{37}{20}}$$

(b) $\frac{xz}{y - z}$

$$\frac{\left(\frac{4}{3}\right)\left(-\frac{3}{4}\right)}{\frac{12}{5} - \left(-\frac{3}{4}\right)} = \frac{-1}{\frac{48 + 15}{20}} = \boxed{-\frac{20}{63}}$$

(c) $\sqrt{3(x - z)}$

$$= \sqrt{3\left(\frac{4}{3} - \left(-\frac{3}{4}\right)\right)}$$

$$= \sqrt{\cancel{3} \cdot \frac{25}{\cancel{12}_{4}}} = \sqrt{\frac{25}{4}} = \boxed{\frac{5}{2}}$$

5. (4 points) Suppose 10 grapes weigh 3 ounces. How much do 18 grapes weigh?

$$\frac{\text{OUNCES}}{\text{GRAPES}} : \quad \frac{3}{10} = \frac{x}{18} \longrightarrow 54 = 10x$$

$$\frac{54}{10} = x \longrightarrow x = \boxed{\frac{27}{5} \text{ or } 5.4 \text{ oz}}$$

6. (4 points) The price of a stock increases by 40% to end the day at \$63. What was the price of the stock before the increase?

$$\frac{\text{START}}{\text{END}} : \quad \frac{x}{63} = \frac{100}{140} \longrightarrow 140x = 6300$$

$$x = \frac{6300}{140} = \frac{14 \cdot 45}{14}$$

$$x = \boxed{\$45}$$

7. (8 points) Simplify the following expressions and eliminate any negative exponents.

(a) $\sqrt{27} + \sqrt{75}$

$$= \sqrt{9}\sqrt{3} + \sqrt{25}\sqrt{3} = 3\sqrt{3} + 5\sqrt{3} = \boxed{8\sqrt{3}}$$

(b) $\sqrt{96x^6y^7z^8}$

$$= \sqrt{16x^6y^6z^8}\sqrt{6y} = \boxed{4x^3y^3z^4\sqrt{6y}}$$

(c) $\left(\frac{3x^3y^{-2}}{4x^{-3}y^2}\right)^{-2}$

$$= \frac{3^{-2}x^{-6}y^4}{4^{-2}x^6y^{-4}} = \frac{4^2}{3^2} \cdot x^{-6-6}y^{4-(-4)}$$

$$= \boxed{\frac{16y^8}{9x^{12}}}$$

8. (16 points) Simplify the following polynomial expressions.

(a) $(2x^2+3x-2)(3x^2-5x-4)$

$= 6x^4 - 10x^3 - 8x^2 + 9x^3 - 15x^2 - 12x - 6x^2 + 10x + 8$

$= \boxed{6x^4 - x^3 - 29x^2 - 2x + 8}$

(b) $x^2-(x+2)(x+3)-(1-x)^2$

$x^2 - (x^2+5x+6) - (1-2x+x^2)$

$x^2 - x^2 - 5x - 6 - 1 + 2x - x^2$

$\boxed{-x^2 - 3x - 7}$

(c) $\left(x+\frac{1}{2}\right)^3 = \left(x+\frac{1}{2}\right)^2\left(x+\frac{1}{2}\right)$

$= \left(x^2 + x + \frac{1}{4}\right)\left(x+\frac{1}{2}\right)$

$= x^3 + \frac{1}{2}x^2 + x^2 + \frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}$

$= \boxed{x^3 + \frac{3}{2}x^2 + \frac{3}{4}x + \frac{1}{8}}$

(d) $\dfrac{35p^{10}q^7 - 45p^5q^6 + 20p^8q^9}{5p^2q^3}$

$= \dfrac{35p^{10}q^7}{5p^2q^3} - \dfrac{45p^5q^6}{5p^2q^3} + \dfrac{20p^8q^9}{5p^2q^3}$

$= \boxed{7p^8q^4 - 9p^3q^3 + 4p^6q^6}$

9. (12 points) Factor the following polynomials completely.

(a) $2x^4 - 8x^3 - 64x^2$

$2x^2(x^2 - 4x - 32) = \boxed{2x^2(x-8)(x+4)}$

(b) $49a^4b^6 - 36x^2y^4$ $= (7a^2b^3)^2 - (6xy^2)^2$

$= \boxed{(7a^2b^3 + 6xy^2)(7a^2b^3 - 6xy^2)}$

(c) $25x^2 - 40xy + 16y^2$ $= (5x)^2 - 2(5x)(4y) + (4y)^2$

$= \boxed{(5x - 4y)^2}$

10. (8 points) Solve the following linear equations for x.

(a) $2(5-2x)+3=x-2$

$10 - 4x + 3 = x - 2$

$+4x \quad +2 \qquad +4x + 2$

$\frac{15}{5} = \frac{5x}{5}$

$\boxed{x = 3}$

(b) $\frac{5}{2x-1} = \frac{2}{3x+4}$

$5(3x+4) = 2(2x-1)$

$15x + 20 = 4x - 2$

$11x = -22$

$\boxed{x = -2}$

11. (12 points) Solve each of the following nonlinear equations for x.

(a) $6x^3 + 60x = x^3 + 40x^2$

$5x^3 - 40x^2 + 60x = 0$

$5x(x^2 - 8x + 12) = 0$

$5x(x-6)(x-2) = 0$

$5x = 0 \qquad x - 6 = 0 \qquad x - 2 = 0$

$x = 0 \qquad x = 6 \qquad x = 2$

$\boxed{x = 0, 2, 6}$

(b) $3x^2 + 5 = 1 - 7x$

$3x^2 + 7x + 4 = 0$

$3x^2 + 3x + 4x + 4 = 0$

$3x(x+1) + 4(x+1) = 0$

$(3x+4)(x+1) = 0$

$3x + 4 = 0$ $\qquad$ $x + 1 = 0$

$3x = -4$ $\qquad$ $x = -1$

$x = -\frac{4}{3}$

$\boxed{x = -\frac{4}{3}, -1}$

(c) $x^3 - 4x^2 - x = -4$

$x^3 - 4x^2 - x + 4 = 0$

$x^2(x-4) - 1(x-4) = 0$

$(x^2 - 1)(x-4) = 0$

$(x+1)(x-1)(x-4) = 0$

$x + 1 = 0$ $\qquad$ $x - 1 = 0$ $\qquad$ $x - 4 = 0$

$x = -1$ $\qquad$ $x = 1$ $\qquad$ $x = 4$

$\boxed{x = -1, 1, 4}$

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12. (5 BONUS points) Suppose a particular store charges its customers a 5% surcharge for using a credit card, but your credit card company gives you a 5% discount on all purchases made at this store. In this situation, is it better to pay with cash or with a credit card? Or does it not matter? Explain your reasoning.

PURCHASE $\xrightarrow[\text{SURCHARGE}]{\text{INCR. } 5\%}$ $\left(1 + \frac{5}{100}\right)$ PURCHASE $\xrightarrow[\text{DISCOUNT}]{\text{DECR } 5\%}$ $\left(1 - \frac{5}{100}\right)\left(1 + \frac{5}{100}\right)$ PURCHASE

MULTIPLIED BY $\frac{95}{100} \cdot \frac{105}{100} = \frac{19}{20} \cdot \frac{21}{20} = \frac{399}{400}$

IT IS BETTER TO PAY WITH CREDIT CARD BECAUSE YOU WILL END UP PAYING ONLY $\frac{399}{400}$ OF THE PURCHASE PRICE (i.e. YOU END UP WITH $\frac{1}{400}^{TH}$ DISCOUNT (i.e. 0.25% DISCOUNT))