Exponents

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Definition of Integer Exponents

When n is a positive integer, by definition,

$$a^n = a \cdot a \cdot \ldots \cdot a \ (n \text{ times}).$$

Notice that when n is increased by one, the result is multiplied by a. Similarly, when n is decreased by one, the result is divided by a, i.e. multiplied by $1/a$. Thus, we can extend the definition of exponents very naturally to 0 and to negative integers as well.

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

Note, in particular, that $a^{-1} = \dfrac{1}{a}$ and $1/a^{-1} = a$.

Practice: p. 428 #67-90

Rules of Exponents

The following properties of exponents follow from the definitions above.

1. $a^m a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}$
3. $(ab)^n = a^n b^n$
4. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
5. $(a^m)^n = a^{mn}$
6. $\left(\dfrac{x^a y^b}{z^c}\right)^n = \dfrac{x^{an} y^{bn}}{z^{cn}}$

The last property listed above is really just a summary of all previous rules, and so is called The General Rule for Exponents (by our textbook).

Practice: p. 430-431