Ratio, Proportion, & Percent

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Ratio

A *ratio* is a comparison of two like quantities. Ratios are fractions.

The ratio of a to b, sometimes denoted $a : b$, is $\dfrac{a}{b}$.

The ratio of b to a, sometimes denoted $b : a$, is $\dfrac{b}{a}$.

When possible, we use the same units for both the numerator and denominator. Examples: Aspect ratios of photos and videos, scales of maps and models, proportions of ingredients.

A *rate* is a comparison of two unlike quantities. Rates are also fractions.

Practice: p. 127-128

Prioportion Problems

A *proportion* is a statement that two ratios or rates are equal. The following are all equivalent.

$$\text{the ratio of } a \text{ to } b \text{ is the same as the ratio of } c \text{ to } d$$

$$a \text{ is to } b \text{ as } c \text{ is to } d$$

$$a : b = c : d$$
$$\frac{a}{b} = \frac{c}{d}$$

Written as above, a andy d are called the *extremes*, while b and c are called to *means*.

In any proportion, the product of the extremes equals the product of the means.

Thus, when three terms of a proportion are known, the fourth term can always be found.

Practice: p. 133-134

Word Problems Using Proportions

1. Represent the unknown quantity by a letter.

2. Set up a proportion, with the unit of measure next to each number in the proportion. The same units must occupy corresponding positions in the two ratios of your proportion. (Think about the cross-product of units.)

3. Solve for the unknown quantity by cross-multiplying.

Practice: p. 136

Percent

By definition,
$$x\% = \frac{x}{100}.$$
Note that it is very easy to convert between decimals and percents.

$$a \text{ (decimal)} = \frac{100 \cdot a}{100} = 100 \cdot a\%$$

Thus to convert a decimal to a percent, move the decimal point two spaces to the right. To convert a percent to a decimal, move the decimal point two spaces to the left.

Practice: p. 138-141

Finding a Fractional Part of a Number

To find a fractional or decimal part of a number (e.g. 5/6 of 48 or 0.8 of 35), simply multiply the two numbers.

Percents are different. Percents are useful because they provide a common language for expressing data, 100 is a convenient denominator to work with, and with time we develop an intuition for thinking about percents (visualizing x out of 100). But when making any sort of calculations involving percents, we must first convert all percents to either fractions or decimals. We do not "do math" with percents!

Practice: p. 143

Percent Problems

$$A \text{ is } P \text{ percent of } B \ \left| \ A = \frac{P}{100} \cdot B \right.$$

$$\left| \ \frac{A}{B} = \frac{P}{100} \right.$$

P precedes "percent". B follows "of". A is the remaining number.

Practice: p.146-147

Word Problems Using Percent

$$\text{percent change decimal} \;=\; \frac{\text{new amount} - \text{original amount}}{\text{original amount}}$$

$$\text{percent change decimal} \;=\; \frac{\text{amount of change}}{\text{original amount}}$$

Note that the percent change decimal is negative when the new amount is less than the original amount, i.e. when the amount of change is negative.

If we let $p =$ percent change decimal, $x_0 =$ original amount, and $x_1 =$ new amount, then

$$p = \frac{x_1 - x_0}{x_0}$$

$$px_0 + x_0 = x_1$$

$$(1 + p)x_0 = x_1$$

The last line above says, for example, that when 124 is *increased* by 25% the result is $(1 + 0.25) \cdot 124 = 1.25 \cdot 124 = 155$, and when 124 is *decreased* by 25% the result is $(1 - 0.25) \cdot 124 = 0.75 \cdot 124 = 93$.

Practice: p. 150-151