$\int \sin^m x \cos^n x \, dx$

i) IF m ODD : $\int (\text{ALL } \cos) \sin x \, dx$

ii) IF n ODD : $\int (\text{ALL } \sin) \cos x \, dx$

iii) IF BOTH EVEN : IDENTITIES.

$\int \sin^m x \cos^n x \, dx$

m, n BOTH EVEN

$\sin^2 x + \cos^2 x = 1$

$\sin 2x = 2 \sin x \cos x \longrightarrow \sin x \cos x = \frac{1}{2} \sin 2x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x \longrightarrow \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$= 2\cos^2 x - 1 \longrightarrow \cos^2 x = \frac{1}{2}(1 + \cos 2x)$

} "HALF ANGLE FORMULAS"

ex. $\int \sin^2 x \, dx = \frac{1}{2} \int 1 - \cos 2x \, dx = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

ex. $\int \sin^2 x \cos^2 x \, dx = \int (\sin x \cos x)^2 \, dx = \frac{1}{4} \int \sin^2 2x$

$= \frac{1}{8} \int 1 - \cos 4x \, dx = \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right) + C$

ex. $\int \cos^4 x \, dx = \int (\cos^2 x)^2 \, dx = \frac{1}{4} \int (1 + \cos 2x)^2 \, dx$

$= \frac{1}{4} \int 1 + 2\cos 2x + \underbrace{\cos^2 2x}_{\frac{1}{2}(1 + \cos 4x)} \, dx$

$= \frac{1}{4} \int \frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x \, dx$

$= \frac{1}{4}\left(\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x\right) + C$

$$\int \tan^m x \sec^n x \, dx$$

1) IF n IS EVEN : $\int (\text{ALL TAN}) \sec^2 x \, dx$ ex. $\int \tan^2 x \sec^4 x \, dx$

2) IF m IS ODD : $\int (\text{ALL SEC}) \sec x \tan x \, dx$ ex. $\int \tan^3 x \sec^3 x \, dx$

3) IF m EVEN, n ODD e.g. $\int \tan^2 x \sec x \, dx$ (HARD!)

$$= \int (\sec^2 x - 1) \sec x \, dx = \int \sec^3 x \, dx - \underbrace{\int \sec x \, dx}_{*}$$

$$\ln \left| \sec x + \tan x \right| + C$$

* $u = \sec x \qquad v = \tan x$

$du = \sec x \tan x \, dx \qquad dv = \sec^2 x \, dx$

$$uv - \int v \, du = \sec x \tan x - \int \underbrace{\tan^2 x}_{\sec^2 x - 1} \sec x \, dx$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$\int \tan^2 x \sec x \, dx \qquad u = \tan x \qquad v = \sec x$

$du = \sec^2 x \, dx \qquad dv = \tan x \sec x \, dx$

$$= \tan x \sec x - \int \sec^3 x \, dx$$