§ 8.2 INTEGRATION BY PARTS

LET $u(x)$, $v(x)$ BE DIFFERENTIABLE FUNCTIONS OF x.

PRODUCT RULE $\Rightarrow$ $$\frac{d}{dx}\left[u(x)\,v(x)\right] = u'(x)\,v(x) + u(x)\,v'(x)$$

(F.T.C.) $$u(x)\,v(x) = \int u'(x)\,v(x)\,dx + \int u(x)\,v'(x)\,dx$$

REARRANGE $$\boxed{\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int v(x)\,u'(x)\,dx}$$

i.e. $$\boxed{\int u\,dv = u\,v - \int v\,du}$$

THE POINT: ONE INTEGRAL $\overbrace{\int u(x)\,v'(x)\,dx}^{\text{OF A PRODUCT OF FUNCTIONS}}$ IS REPLACED WITH ANOTHER INTEGRAL OF A <u>DIFFERENT</u> PRODUCT OF FUNCTIONS.

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$$\int v(x)\,u'(x)\,dx$$

(HOPEFULLY SIMPLER)

<u>ex 1.</u> $\int x^3 \ln x \, dx$

<u>RULE OF THUMB:</u> CHOOSE u TO BE A FUNCTION THAT CAN BE DIFFERENTIATED REPEATEDLY,

v' TO BE A FUNCTION THAT CAN BE INTEGRATED REPEATEDLY,

WITHOUT MUCH DIFFICULTY,

POSSIBLY PRODUCING SIMPLER FUNCTIONS

ex 2. $\int x^2 \sin x \, dx$ (2 I.B.P.'s)

ex 3. $\int \ln x \, dx$

ex 4. $\int \tan^{-1} x \, dx$

$u = \tan^{-1} x$ $\quad v = x$

$du = \frac{1}{1+x^2} dx$ $\quad dv = dx$

$$= x \tan^{-1} x - \int \frac{x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \ln|1+x^2| + C$$

(FOR DEFINITE INTEGRALS, F.T.C. PART II $\Rightarrow$

$$\int_a^b u \, dv = uv\Big|_a^b - \int_a^b v \, du$$)

ex. $\int_0^1 x^5 e^{x^3} dx$

FIRST: $w = x^3$

$dw = 3x^2 dx$

$$\frac{1}{3} \int_0^1 w e^w \, dw$$

ex. $\int e^x \sin x \, dx$ (STICK TO YOUR GUNS! SOLVE ALGEBRAICALLY)

ex. $\int x^2 \sin x^3 \, dx$ (u-SUB FIRST)

REDUCTION FORMULAS:

$$\int \sin^n x \, dx \qquad \text{let} \quad u = \sin^{n-1} x \qquad v = -\cos x$$

$$du = (n-1)\sin^{n-2} x \cos x \, dx \qquad dv = \sin x \, dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \overbrace{\cos^2 x}^{1-\sin^2 x} dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \left[\int \sin^{n-2} x \, dx - \int \sin^n x \, dx \right]$$

$$n \int \sin^n x = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \, dx$$

$$\therefore \boxed{\int \sin^n x = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx}$$

SIMILARLY,
$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$