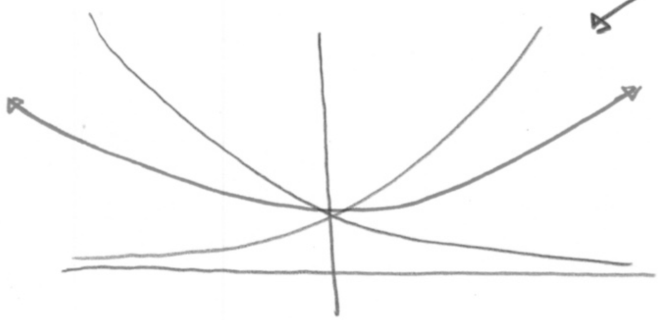
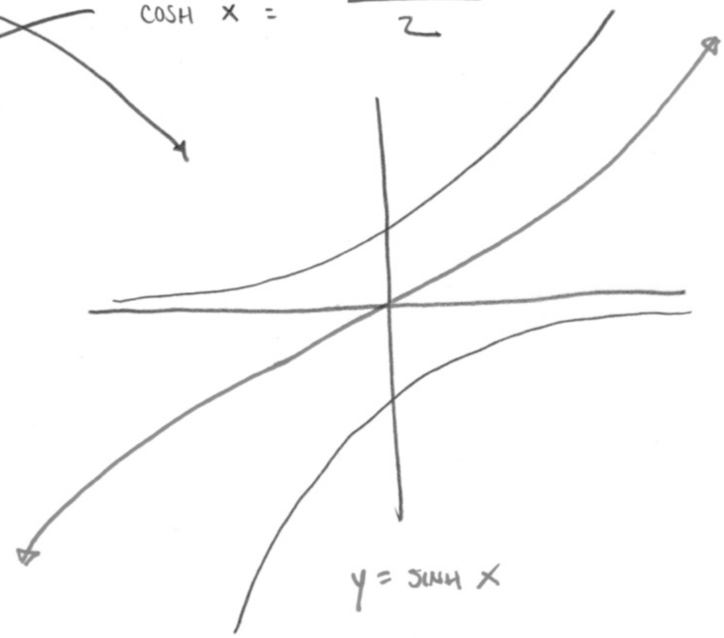
§7.3 Hyperbolic Functions

Def: $\sinh x = \dfrac{e^x - e^{-x}}{2}$ $\qquad$ $\cosh x = \dfrac{e^x + e^{-x}}{2}$



$y = \cosh x$

EVEN



$y = \sinh x$

ODD

Others: $\operatorname{csch} x = \dfrac{1}{\sinh x}$, $\quad \operatorname{sech} x = \dfrac{1}{\cosh x}$

$\tanh x = \dfrac{\sinh x}{\cosh x}$, $\quad \coth x = \dfrac{\cosh x}{\sinh x}$

Identities: $\cosh^2 x - \sinh^2 x = 1$ (check)

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

ex: If $\sinh x = \frac{1}{2}$, find 5 other H-trig values for x.

DERIVATIVES:

$$\frac{d}{dx}\left[\sinh x\right] = \cosh x \qquad \frac{d}{dx}\left[\operatorname{csch} x\right] = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}\left[\cosh x\right] = \sinh x \qquad \frac{d}{dx}\left[\operatorname{sech} x\right] = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}\left[\tanh x\right] = \operatorname{sech}^2 x \qquad \frac{d}{dx}\left[\coth x\right] = -\operatorname{csch}^2 x$$

INTEGRALS ANTI-DERIVATIVES

ex. DIFFERENTIATE $y = \ln \cosh x - \frac{1}{2}\tanh^2 x$

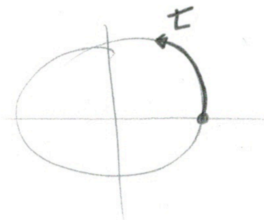
$$y = x^2 \tanh \frac{1}{x}$$

ex. INTEGRATE $\int \sinh \frac{x}{5}\, dx$

$$\int 4 \cosh(3x - \ln 2)\, dx$$

(§7.3)

$$x^2 + y^2 = 1$$

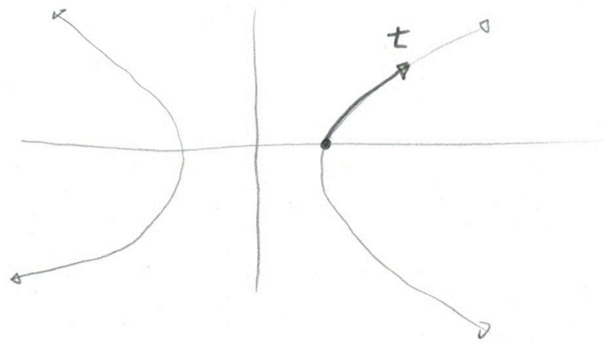


$x = \cos t$

$y = \sin t$

$$\cos^2 t + \sin^2 t = 1$$

$$x^2 - y^2 = 1$$

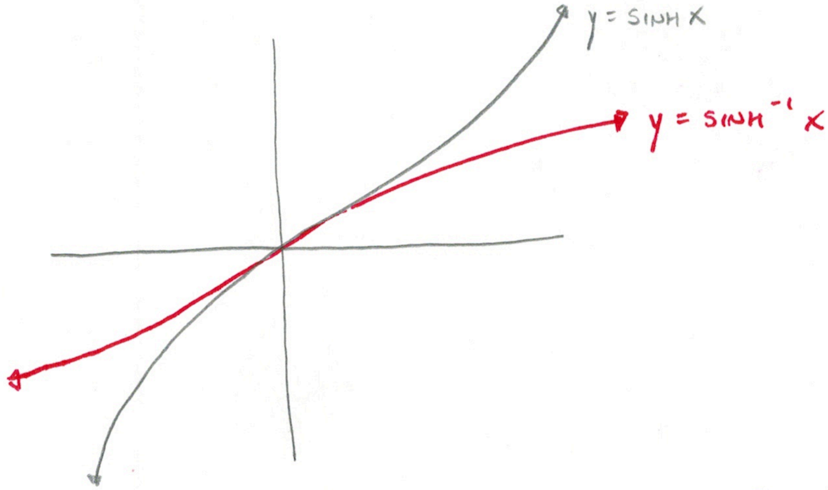


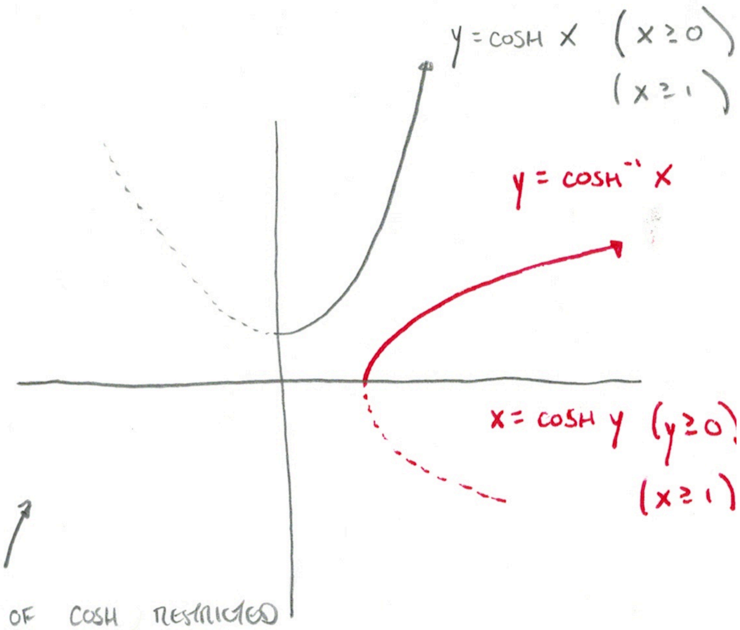
$x = \cosh t$

$y = \sinh t$

$$\cosh^2 t - \sinh^2 t = 1$$

INVERSE HYPERBOLIC FUNCTIONS





DOMAIN OF COSH RESTRICTED TO $[0, \infty)$ TO MAKE IT 1-1 (INVERTIBLE)

(SIMILAR TO HOW $\sin^{-1}$ & $\cos^{-1}$ WERE DEFINED)

$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$$

LET $y = \sinh^{-1} x$. FIND y'.

$$\frac{d}{dx}\left[\sinh y = x\right]$$

$$\cosh y \cdot y' = 1$$

$$y' = \frac{1}{\cosh y} = \frac{1}{\sqrt{1+\sinh^2 y}} = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$$

ex. $$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \frac{1}{a}\int \frac{1}{\sqrt{1+\left(\frac{x}{a}\right)^2}}\,dx \qquad \begin{matrix} u = \frac{x}{a} \\ a\,du = dx \end{matrix} \qquad = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

ex. $$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$$