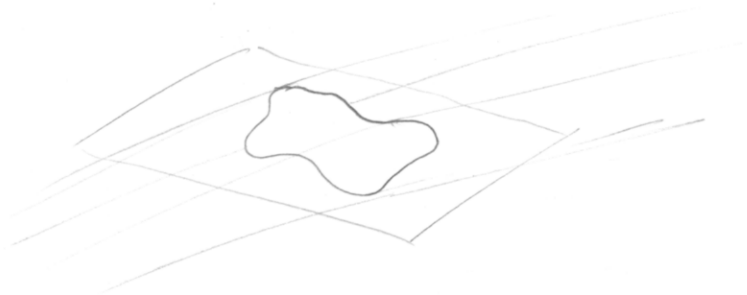


§ 10.6 CYLINDERS & QUADRIC SURFACES

Def A CYLINDER IS A SURFACE THAT CONSISTS OF ALL LINES (CALLED RULINGS)
|| TO A GIVEN LINE & PASSING THROUGH A GIVEN PLANE CURVE.



e.g. $x^2 + y^2 = 1$, $x^2 + z^2 = 1$, $y^2 + z^2 = 1$

$z = x^2 - 2x + 1$, $z = \cos(y)$, $y = f(x)$

EG MISSING $x/y/z$ DESCRIBES CYLINDER WITH RULING || TO $x/y/z$ -AXIS.

QUADRIC SURFACES

2nd DEGREE EQ IN x, y, z .

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

↓
TRANSLATIONS, ROTATIONS

$$Ax^2 + By^2 + Cz^2 + D = 0 \quad \text{or} \quad Ax^2 + By^2 + Cz = 0$$

SKETCH USING CROSS SECTION

$$9x^2 - y^2 + z^2 = 0$$

$$25x^2 + 4y^2 + z^2 = 100$$

$$4x^2 + 9y^2 + z = 0$$

$$z = y^2 - x^2$$

§12.6 CYLINDERS & QUADRIC SURFACES

UNDERSTAND WHAT THE GRAPHS OF EQUATIONS IN 3 VARIABLES LOOK LIKE.

EQ. $x^2 + y^2 - 2z = 3$

GRAPH IS THE SET OF ALL POINTS (x, y, z) THAT SATISFY THE EQ.

$(\sqrt{3}, 0, 0)$ IS ON THE GRAPH ✓

$(1, 2, 5)$ IS NOT ON THE GRAPH (X)

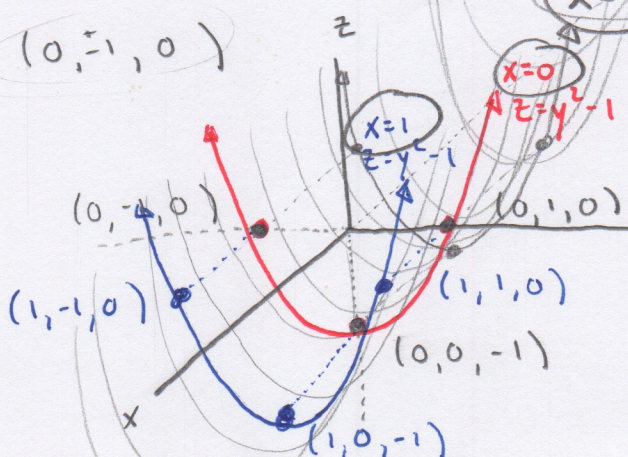
$(0, 0, -\frac{3}{2})$ IS ON THE GRAPH ✓

ex. $z - y^2 + 1 = 0$ *

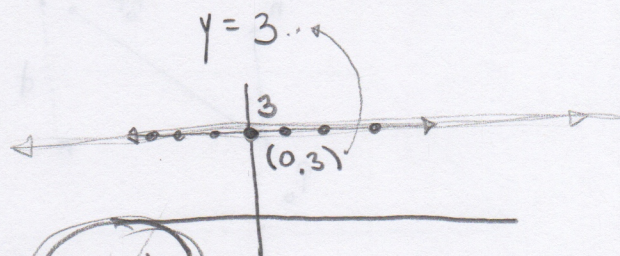
Pld: $(0, 0, -1)$ on GRAPH

$-1 - 0^2 + 1 = 0$ ✓

$(0, 1, 0)$



RECALL: EQ IN 2 VAR.



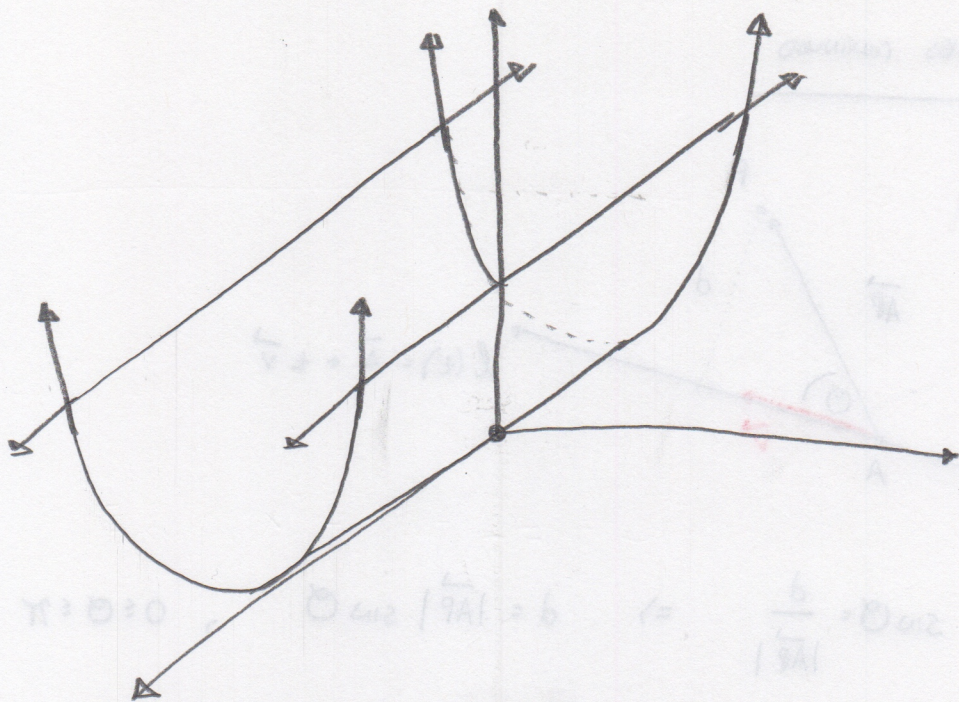
FIX 1 COORD
EQUAL TO A CONSTANT

e.g. FIX $x = 0$.

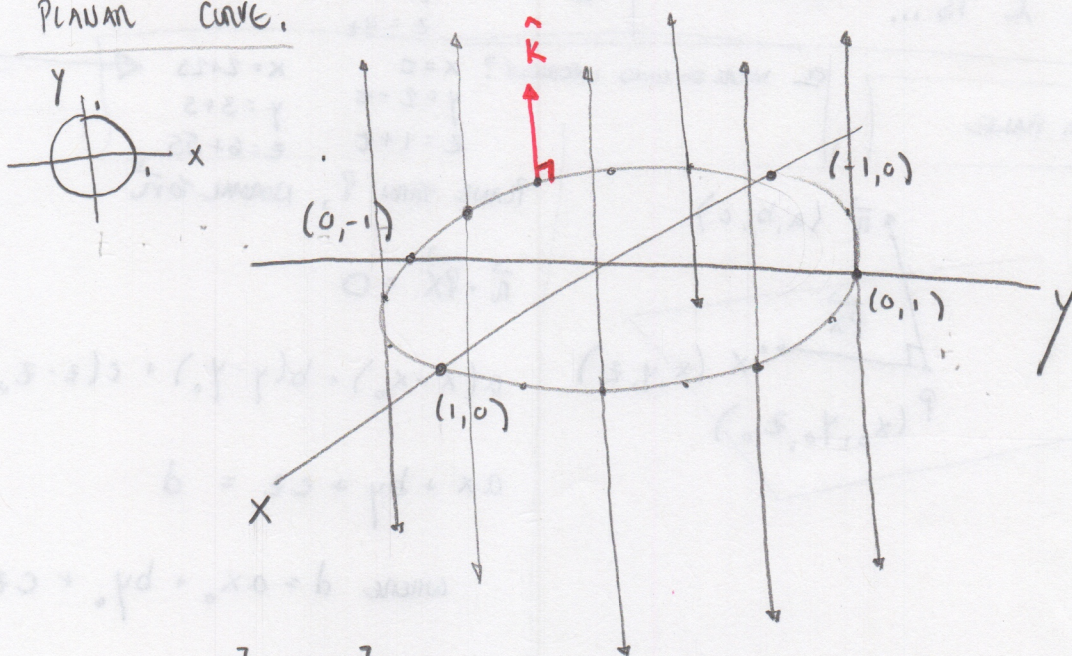
* $z = y^2 - 1$

FIX $x = 1$

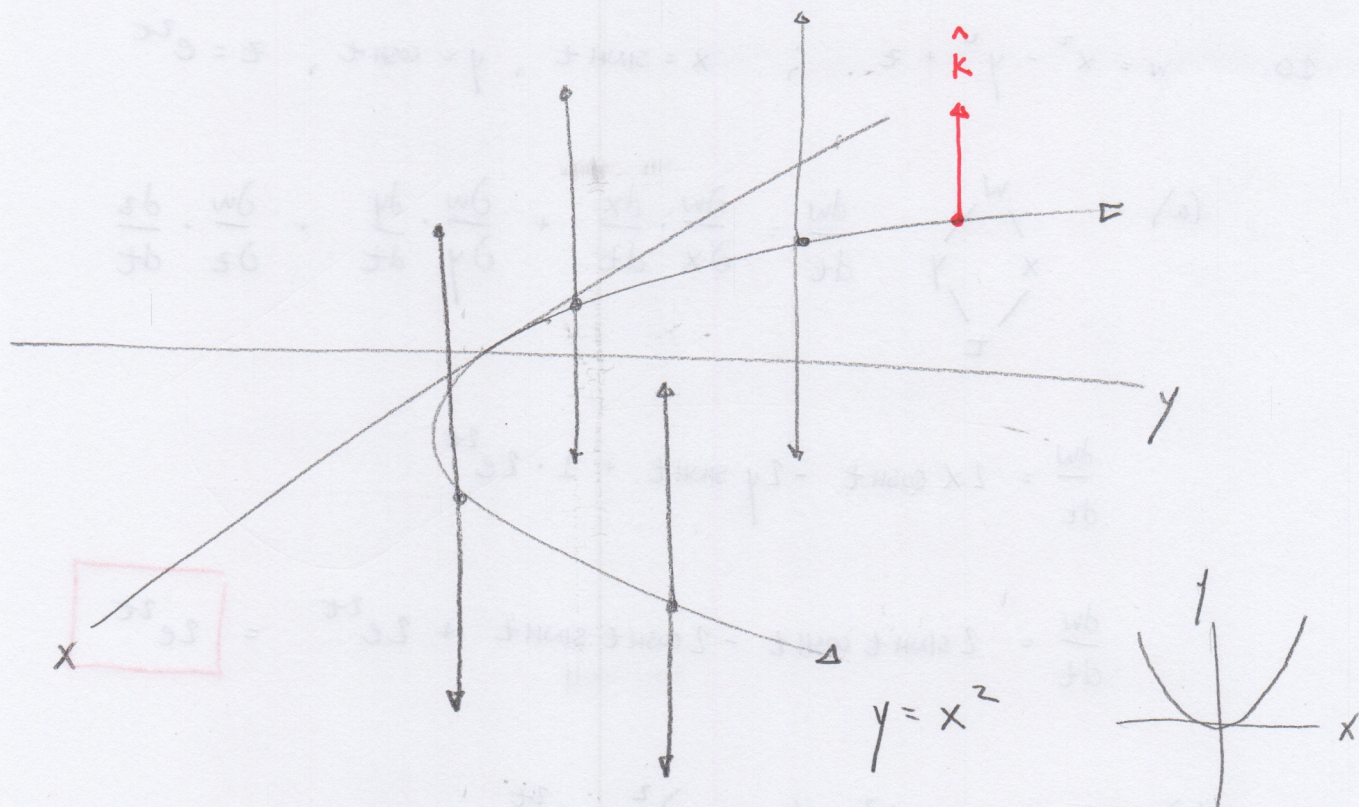
$z = y^2 - 1$



Def: A cylinder is a surface that consists of all lines called rulings parallel to a given vector $\vec{k}$ passing through a given planar curve.



e.g. $x^2 + y^2 = 1$



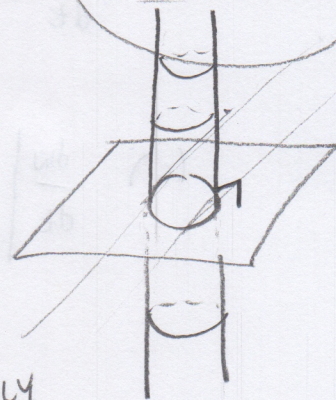
WE WILL FOCUS ON CYLINDERS WHERE THE
PLANAR CURVE LIES IN A COORDINATE PLANE

(i.e. $x=0$, $y=0$, $z=0$)

& THE VECTOR IS $\perp$ TO THAT PLANE

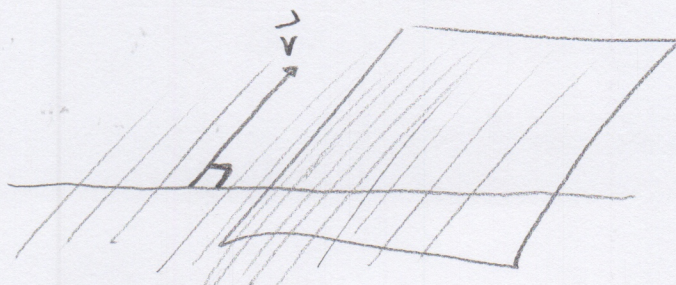
(i.e. $\hat{i}$, $\hat{j}$, $\hat{k}$)

CIRCULAR
RIGHT CYLINDER



THESE CYLINDER HAVE EQ'S INVOLVING ONLY
2 VARIABLES

(i.e. MISSING x , MISSING y , MISSING z)

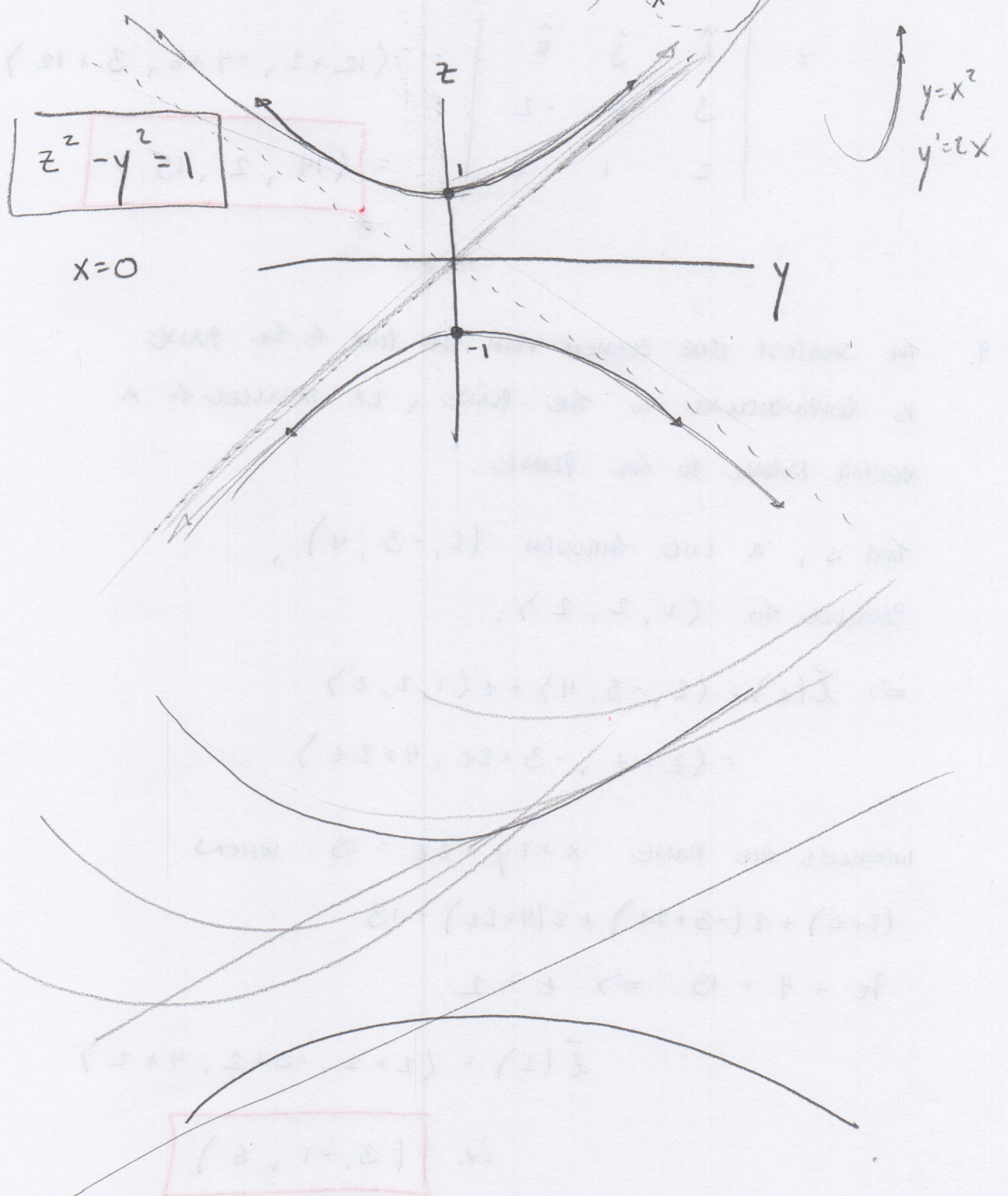


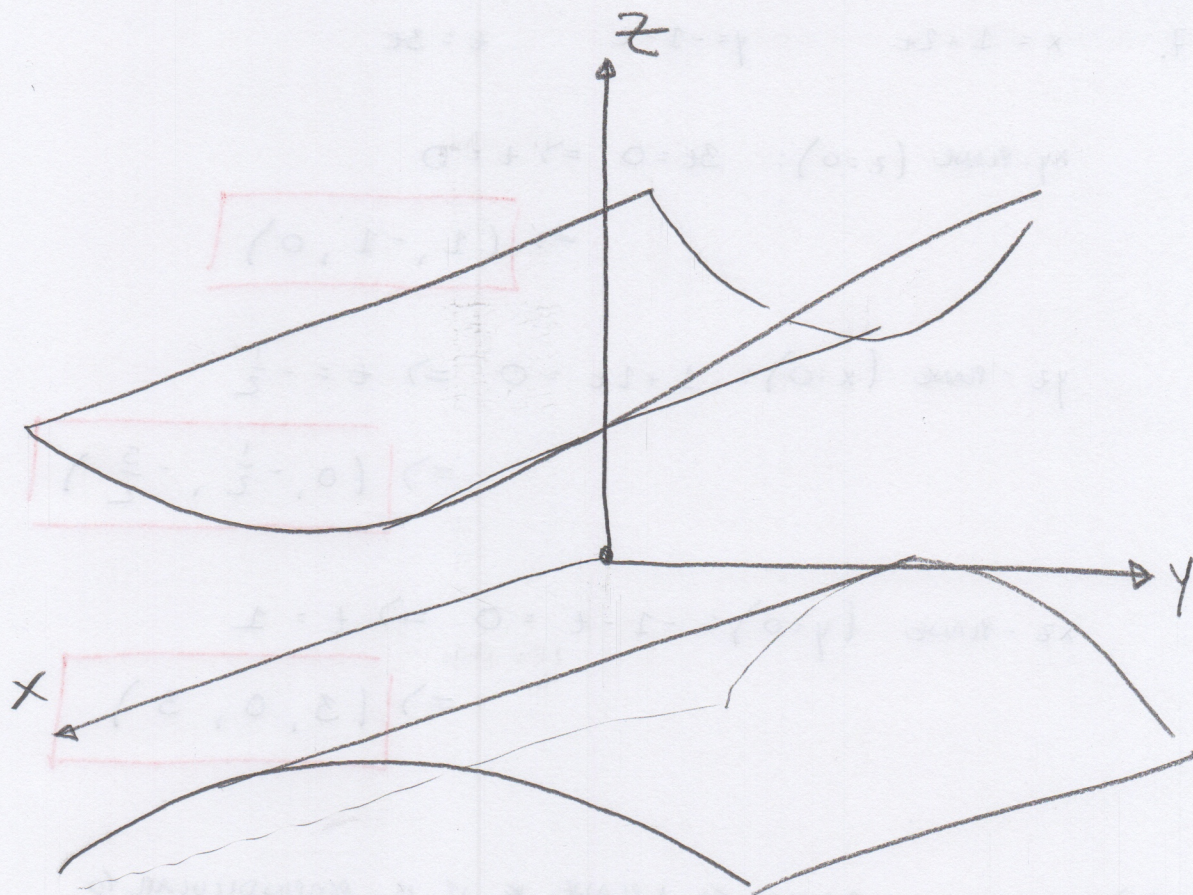
EX.

SKETCH THE GRAPH OF

$$z^2 - y^2 = 1$$

MISSING x .





Def: THE GRAPH OF AN EQUATION IN 3 VARIABLES IS CALLED
A SURFACE.

IF THE EQUATION HAS THE FORM

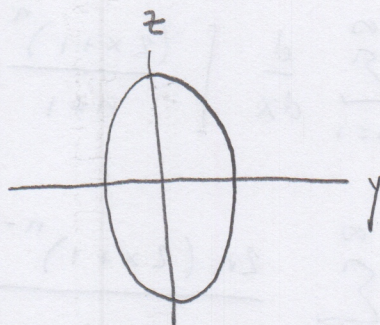
$$\underbrace{Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz}_{2^{\text{nd}} \text{ DEGREE TERMS}} + \underbrace{Gx + Hy + Iz}_{1^{\text{st}} \text{ DEGREE TERMS}} + \underbrace{J}_{0 \text{ DEGREE TERM}} = 0$$

THEN ITS GRAPH (i.e. SURFACE) IS CALLED A
QUADRIC SURFACE.

e.g. SKETCH THE QUADRIC SURFACE:

$$25x^2 + 4y^2 + z^2 = 100$$

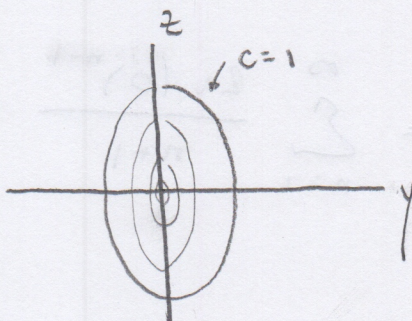
FIX x . $x=0$:



$$4y^2 + z^2 = 100$$

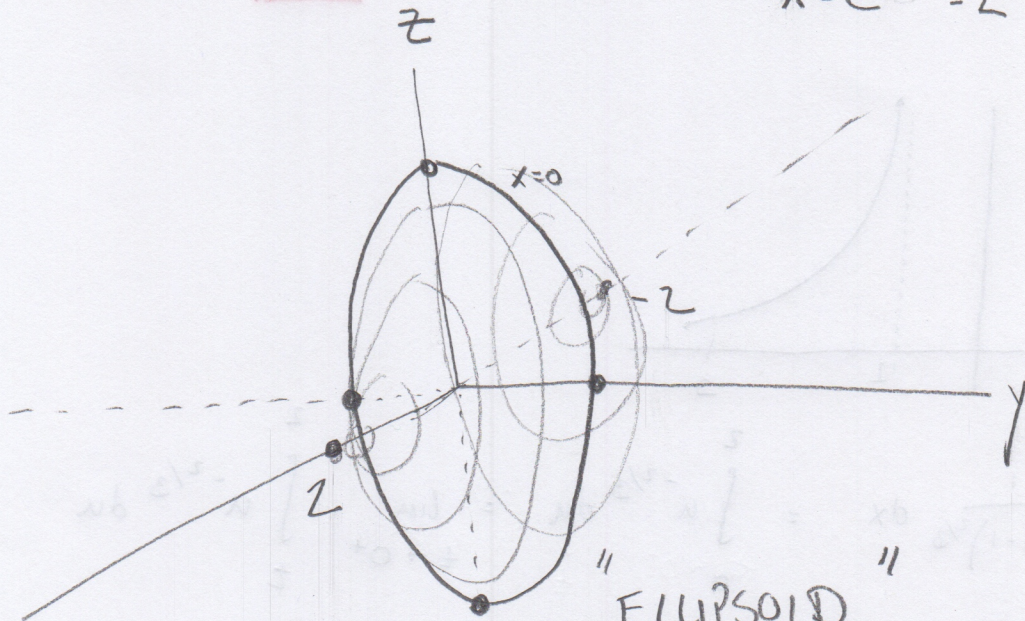
$$c=0$$

$x=c$



$$4y^2 + z^2 = 100 - 25c^2$$

$x=c = \pm 2$



SPHERE $x^2 + y^2 + z^2 = r^2$

ELLIPSOID. $ax^2 + by^2 + cz^2 = r^2$

$$z=1$$

$$1 = x^2 + 16y^2$$

$$x=0$$

$$1 = 16y^2 \rightarrow y = \pm \frac{1}{4}$$

$$y=0$$

$$1 = x^2$$

$$x = \pm 1$$

$$\begin{aligned} &(0, \pm \frac{1}{4}, 1) \\ &(\pm 1, 0, 1) \end{aligned}$$

$$(0, 0, 0)$$

$$[0, 1, 0]$$

§14.1 FUNCTIONS OF SEVERAL VARIABLES

AREA OF A RECTANGLE

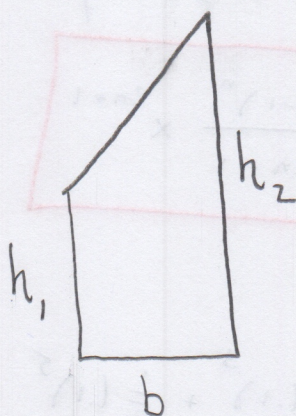
$$A = lw$$

DEPENDENT
VARIABLE

INDEPENDENT
VARIABLES

$$A(l, w) = lw$$

AREA OF A TRAPEZOID



$$A = \frac{1}{2} b (h_1 + h_2)$$

$$A(b, h_1, h_2) = \frac{1}{2} b (h_1 + h_2)$$

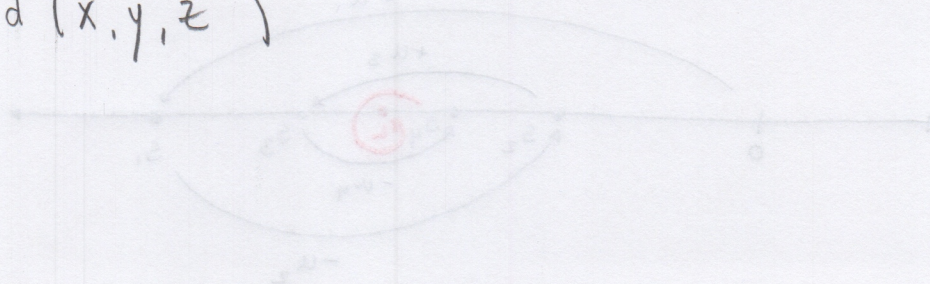
INDEP. VAR.

DEP. VAR.

DISTANCE FROM A POINT (x, y, z) TO $(3, 4, 5)$

$$d = \sqrt{(x-3)^2 + (y-4)^2 + (z-5)^2}$$

$$d(x, y, z)$$



x y
 $''$ $''$

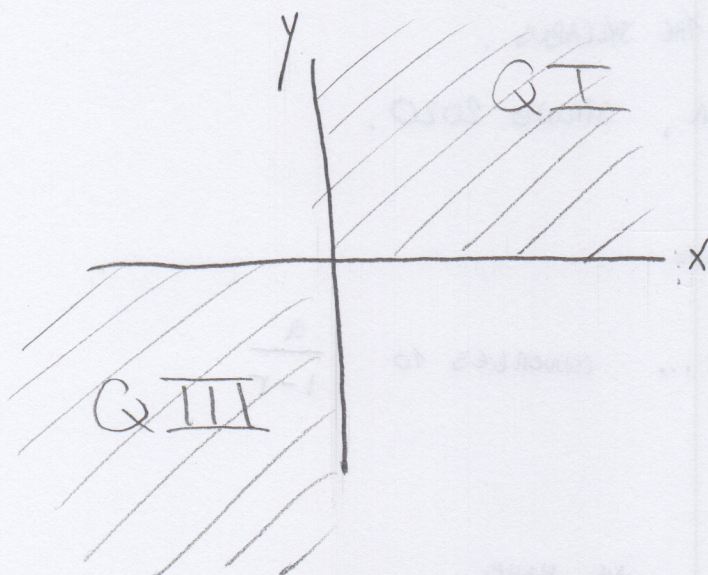
ex. $f(x, y) = \sqrt{xy}$; $f(2, 8) = \sqrt{2 \cdot 8} = 4$

$f(3, 6) = \sqrt{3 \cdot 6} = \sqrt{18} = 3\sqrt{2}$

$\text{Dom}(f) : xy \geq 0 \rightarrow xy = 0 \text{ or } xy > 0$

$\swarrow \quad \searrow$
 $x=0$ $y=0$
 $y\text{-axis}$ $x\text{-axis}$

x, y HAVE
 SAME SIGN



DOMAIN OF $f(x, y)$

IS A SUBSET OF

THE xy -PLANE

CALLED A REGION

ex. $f(x,y) = \frac{\sqrt{y-3x+5}}{x^2-4}$

$f(1,2) = \dots$

$\text{Dom}(f): y-3x+5 \geq 0 \quad x^2-4 \neq 0$

$y \geq 3x-5$

$(x+2)(x-2) \neq 0$

$x \neq \pm 2$

$\text{Dom}(f) = \{ (x,y) \mid y \geq 3x-5, x \neq \pm 2 \}$

Sketch $\text{Dom}(f)$:

