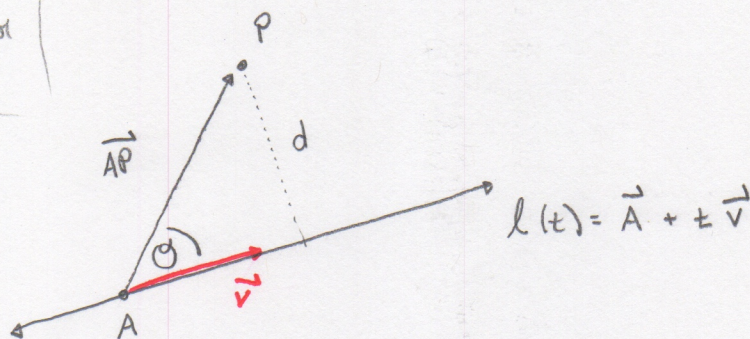


§12.5 LINES & PLANES CONTINUED

DISTANCE FROM A POINT
TO A LINE.



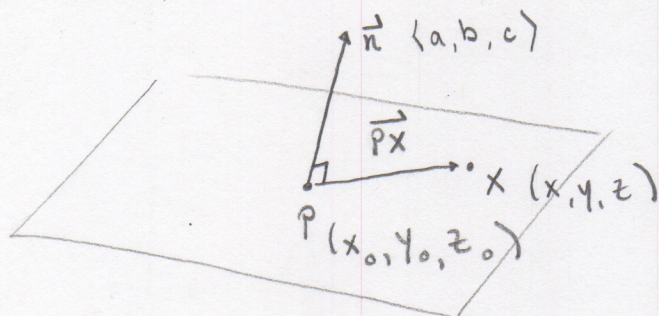
$$\sin \theta = \frac{d}{|\vec{AP}|} \Rightarrow d = |\vec{AP}| \sin \theta, \quad 0 \leq \theta \leq \pi$$

l is a line thru A
parallel to $\vec{v}$. The
distance from a point P
to the line l is ...

$$d = \frac{|\vec{AP}| |\vec{v}| \sin \theta}{|\vec{v}|} = \frac{|\vec{AP} \times \vec{v}|}{|\vec{v}|}$$

ex. line $\begin{matrix} x = 10 + 4t \\ y = -3 \\ z = 4t \end{matrix}$ point $(-1, 4, 3)$

EQUATIONS OF A PLANE



ex. WHERE DO LINES INTERSECT? $\begin{matrix} x = t \\ y = 2 - t \\ z = 1 + t \end{matrix}$ $\begin{matrix} x = 2 + 2s \\ y = 3 + s \\ z = 6 + 5s \end{matrix}$

PLANE THRU P , NORMAL TO $\vec{n}$

$$\vec{n} \cdot \vec{PX} = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz = d$$

$$\text{WHERE } d = ax_0 + by_0 + cz_0$$

Def: A PLANE IS NORMAL TO VECTOR (& VICE VERSA)

WHEN THE TWO ARE PERPENDICULAR.

ex. PLANE THRU $(2, 4, 5), (1, 5, 7), (-1, 6, 8)$

ex. PLANE CONTAINING THE LINES

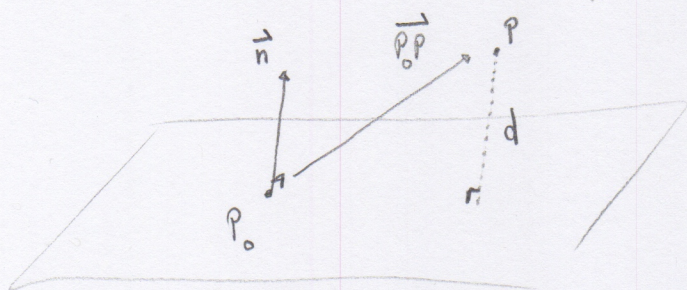
ex. FIND THE POINT WHERE THE LINE $x = 2$ MEETS THE PLANE $6x + 3y - 4z = -12$
 $y = 3 + 2t$
 $z = -2 - 2t$

ex. PARAMETERISE THE LINE IN WHICH THE PLANES $3x - 6y - 2z = 3$, $2x + y - 2z = 2$ INTERSECT.

ex. IS THE LINE $x = 1 - t$ PARALLEL TO THE PLANE $3x - 7y + 2z = 1$?
 $y = 2 + 4t$
 $z = 3 - 3t$
 $x + y + z = -1$?

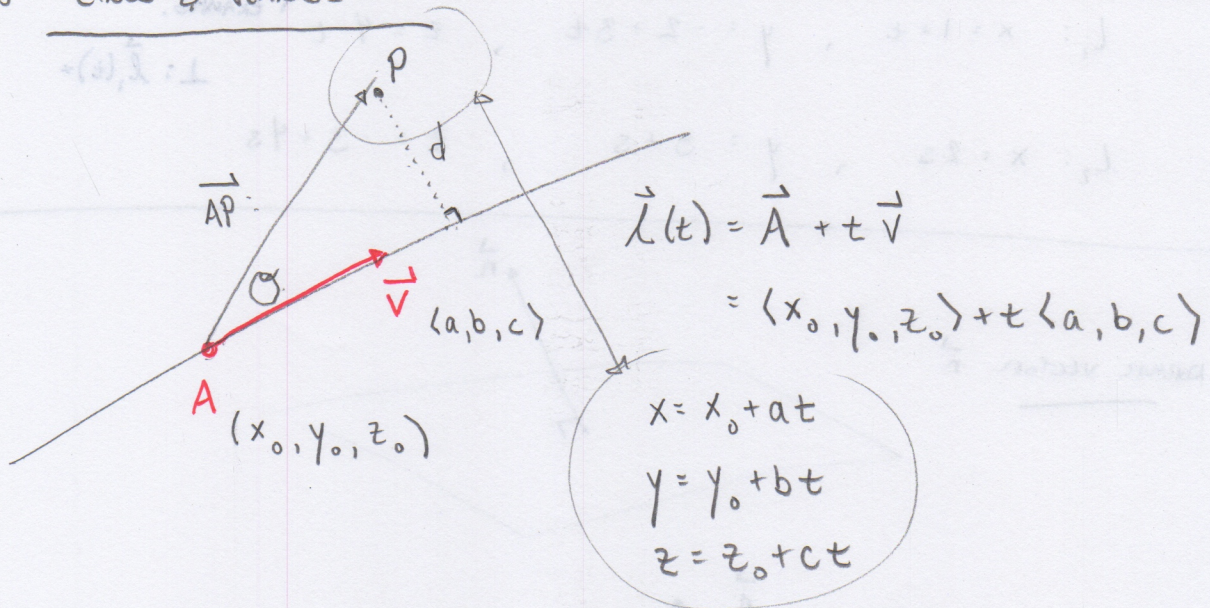
Def. ANGLE BETWEEN PLANES = ANGLE BETWEEN NORMAL VECTORS.

DISTANCE FROM POINT TO PLANE



$$d = \left| \text{comp}_{\vec{n}} \vec{P_0P} \right| = \left| \frac{\vec{P_0P} \cdot \vec{n}}{|\vec{n}|} \right|$$

§ 12.5 LINES & PLANES



$$\sin \theta = \frac{d}{|\vec{AP}|} \Rightarrow d = |\vec{AP}| \sin \theta$$

$$d = \frac{|\vec{AP}| |\vec{v}| \sin \theta}{|\vec{v}|} = \frac{|\vec{AP} \times \vec{v}|}{|\vec{v}|}$$

ex.

DISTANCE?

LINE:

$$x = 10 + 4t$$

$$y = -3 + 0t$$

$$z = 4t$$

Point: $(-1, 4, 3)$

$$\vec{AP} = \langle -11, 7, 3 \rangle$$

Point on line

$$A(10, -3, 0)$$

($t=0$)

$$\vec{v} = \langle 4, 0, 4 \rangle$$

$$\vec{r}(t) = \langle 10, -3, 0 \rangle + t\langle 4, 0, 4 \rangle$$

$$\text{DISTANCE}(P, \ell) = \frac{|\vec{AP} \times \vec{v}|}{|\vec{v}|}$$

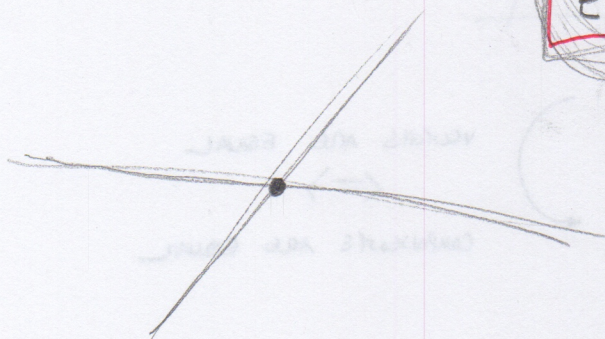
$$= \frac{\sqrt{28^2 + 56^2 + 28^2}}{\sqrt{4^2 + 0^2 + 4^2}}$$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -11 & 7 & 3 \\ 4 & 0 & 4 \end{vmatrix} = \langle 28, 56, -28 \rangle$$

ex. WHERE DO LINES
INTERSECT?

$$\begin{aligned} x &= t \\ y &= 2 - t \\ z &= 1 + t \end{aligned}$$

$$\begin{aligned} x &= 2 + 2t = 2 + 2s \\ y &= 3 + t = 3 + s \\ z &= 6 + 5t = 6 + 5s \end{aligned}$$



USE DIFFERENT PARAMETERS t, s
SET COMPONENTS EQUAL TO SOLVE
SYSTEM OF EQ.

$$x: t = 2 + 2s$$

$$y: 2 - t = 3 + s$$

$$z: 1 + t = 6 + 5s$$

$$2 - (2 + 2s) = 3 + s$$

$$-2s = 3 + s$$

$$-3 = 3s$$

$$s = -1$$

$$t = 0$$

$$x = 0$$

$$y = 2$$

$$z = 1$$

$$(0, 2, 1)$$

$$x = 2 + 2s \rightarrow 0$$

$$y = 3 + s \rightarrow 2$$

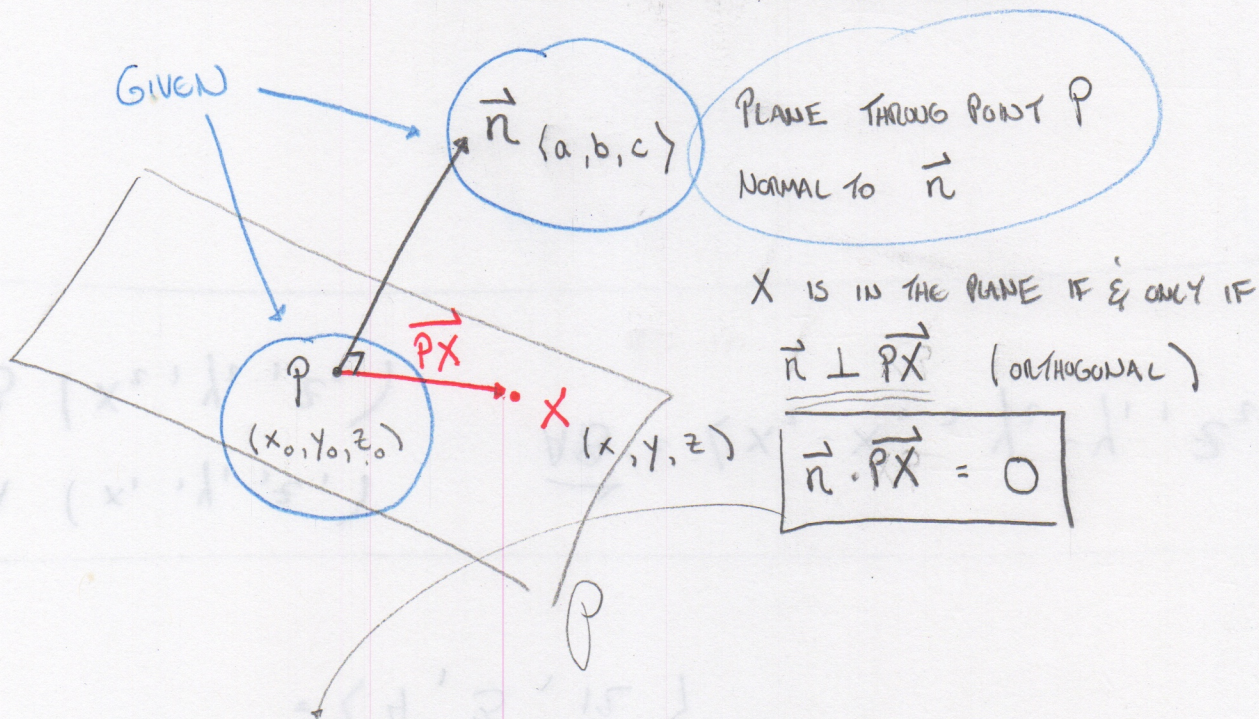
$$z = 6 + 5s \rightarrow 1$$

$$(0, 2, 1)$$

EQUATION OF A PLANE

Def: A PLANE IS NORMAL TO VECTOR (& VICE VERSA)

WHEN THEY ARE PERPENDICULAR TO EACH OTHER.



$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz = d$$

← EQ. OF PLANES
LOOK LIKE THIS!

WHERE $d = ax_0 + by_0 + cz_0$

COEFFICIENTS GIVE $\vec{n} = \langle a, b, c \rangle$

ex. $2x - 5y + 1z = -4$

POINT: $(-2, 0, 0), (0, 0, -4)$

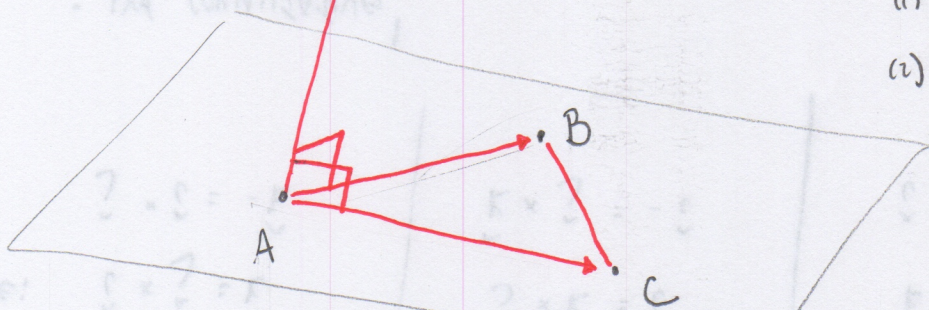
$$\vec{n} = \langle 2, -5, 1 \rangle$$

$$\langle -4, 10, -2 \rangle$$

ex. GIVE EQ. FOR PLANE THRU

$A(2, 4, 5), B(1, 5, 7), C(-1, 6, 8)$

$\vec{n} = \vec{AB} \times \vec{AC}$



(1) POINT IN PLANE P ✓✓✓

(2) Normal vector $\vec{n}$ ✓

$(\vec{n} \cdot \vec{PX} = 0)$

$\vec{AB} = \langle -1, 1, 2 \rangle$

$\vec{AC} = \langle -3, 2, 3 \rangle$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix} = \langle -1, -3, 1 \rangle$ ✓

3-4 -6+3

$\vec{n} \cdot \vec{AX} = 0$ $\times \langle x, y, z \rangle - A \langle 2, 4, 5 \rangle$

$\langle -1, -3, 1 \rangle \cdot \langle x-2, y-4, z-5 \rangle = 0$

$- (x-2) - 3(y-4) + (z-5) = 0$

$-x - 3y + z = -9$

$x + 3y - z = 9$

$2x + 6y - 2z = 18$

ex. FIND POINT WHERE THE LINE

$$x = 2$$

$$y = 3 + 2t$$

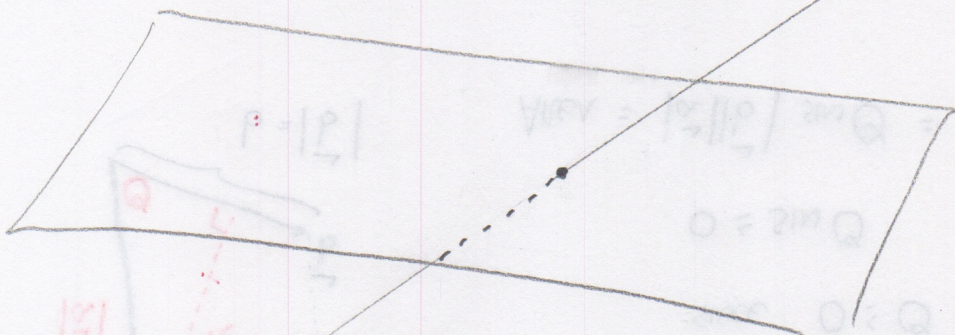
$$z = -2 - 2t$$

$$t = -\frac{41}{14}$$

$$\left(2, 3 - \frac{41}{7}, -2 + \frac{41}{7} \right)$$

MEETS THE PLANE

$$6x + 3y - 4z = -12$$



$$6(2) + 3(3+2t) - 4(-2-2t) = -12$$

$$12 + 9 + 6t + 8 + 8t = -12$$

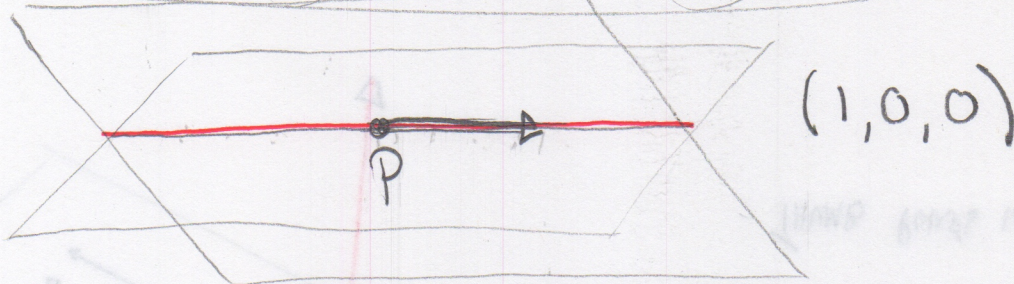
$$14t + 29 = -12$$

$$14t = -41$$

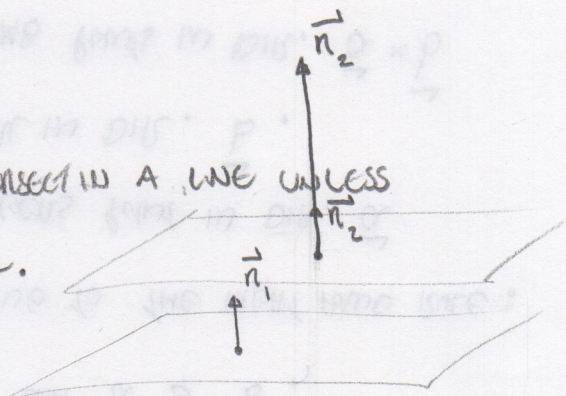
$$t = -\frac{41}{14}$$

ex. PARAMETERIZE THE LINE IN WHICH THE PLANES

$$3x - 6y - 2z = 3, \quad 2x + y - 2z = 2 \quad \text{INTERSECT.}$$

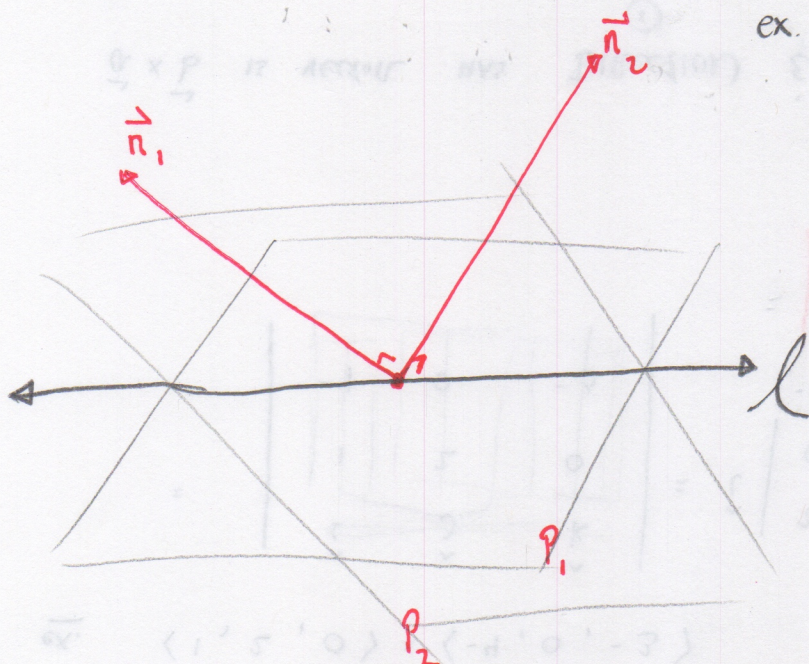


NOTE: ANY 2 PLANES INTERSECT IN A LINE UNLESS THEY ARE PARALLEL.



$$\vec{n}_1 = c \vec{n}_2 \quad (\text{CONSTANT MULTIPLE OF})$$

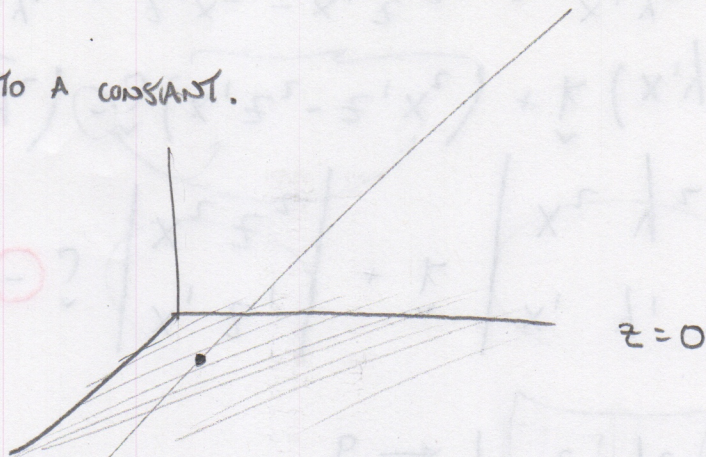
ex. $2x - 5y + 11z = 1$
 $-6x + 15y - 33z = 0$ } PARALLEL



LINE OF INTERSECTION
 (IS IN BOTH PLANES AND)
 IS $\perp$ TO BOTH NORMAL VECTORS
 $\vec{n}_1$ & $\vec{n}_2$.

$$\Rightarrow \text{PARALLEL TO } \vec{n}_1 \times \vec{n}_2$$

Set ONE coord to a constant.



$z=0$

$$3x - 6y = 3$$

$$3x = 3 + 6y$$

$$x = 1 + 3y$$

$$2x + y = 2$$

$$2(1 + 3y) + y = 2$$

$$2 + 6y + y = 2$$

$$7y = 0$$

$$x = 1$$

$$y = 0$$

$(1, 0, 0)$ Point on both lines

Line : Point $(1, 0, 0)$ Direction: $\vec{v} = \vec{n}_1 \times \vec{n}_2$

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = \langle 14, 2, 15 \rangle$$

$$\begin{aligned} \vec{r}(t) &= \langle 1, 0, 0 \rangle + t \langle 14, 2, 15 \rangle \\ &= \langle 1 + 14t, 2t, 15t \rangle \end{aligned}$$

$$x = 1 + 14t, \quad y = 2t, \quad z = 15t$$

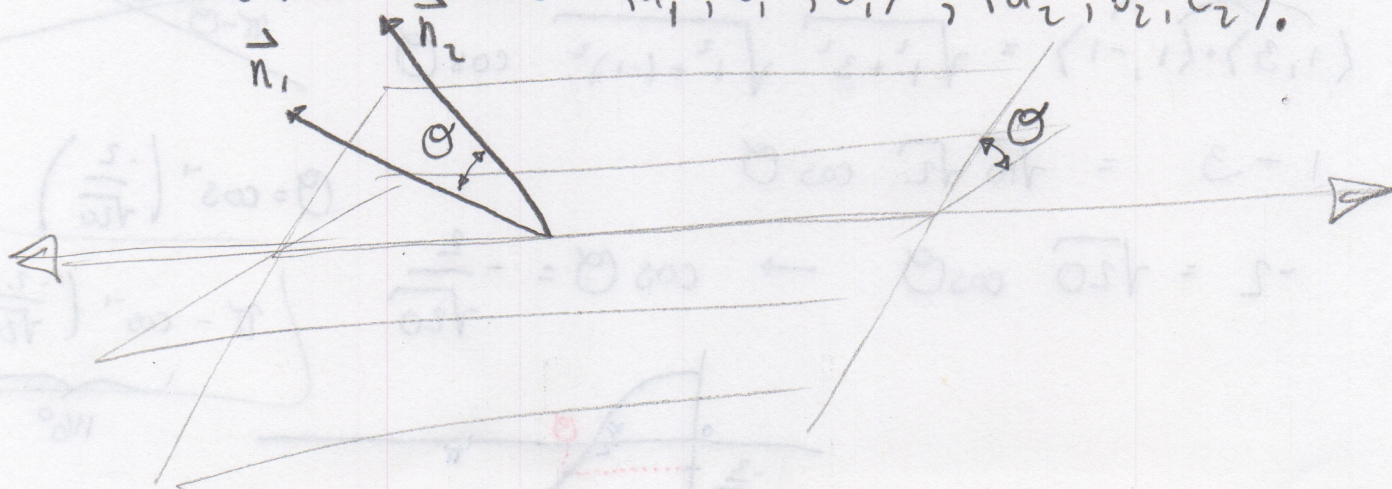
Def: ANGLE BETWEEN 2 PLANES

$$a_1x + b_1y + c_1z = d_1$$

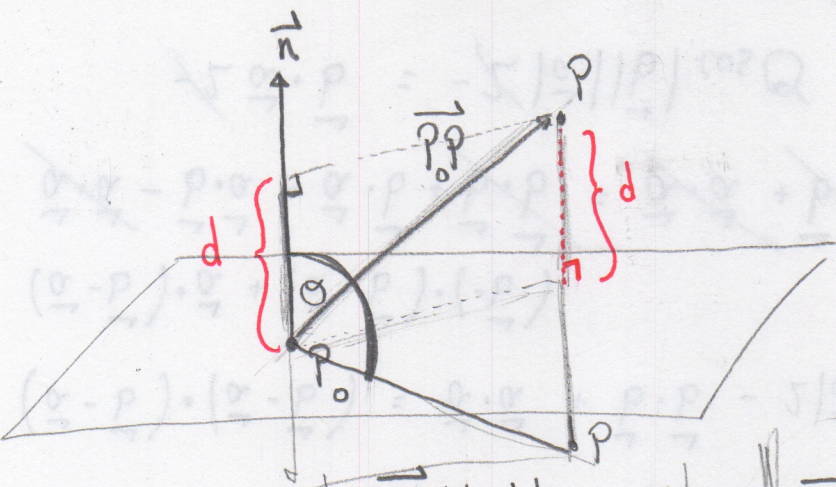
$$a_2x + b_2y + c_2z = d_2$$

IS EQUAL TO THE ANGLE BETWEEN THE

NORMAL VECTORS $\langle a_1, b_1, c_1 \rangle, \langle a_2, b_2, c_2 \rangle$.



DISTANCE FROM POINT P TO A PLANE



$$\cos \theta = \frac{d}{|\vec{P_0P}|}$$

$$d = |\vec{P_0P}| \cos \theta$$

$$d = \frac{|\vec{P_0P}| |\vec{n}| \cos \theta}{|\vec{n}|}$$

$$= \frac{|\vec{P_0P} \cdot \vec{n}|}{|\vec{n}|}$$

comp $\vec{n}$ $\vec{P_0P}$

$$\left(\vec{Q} \text{ is vector from } \vec{P_0} \text{ to } \vec{P} \right)$$

$$\vec{Q} \cdot \vec{P} = |\vec{Q}| |\vec{P}| \cos \theta$$

$$\vec{r_0} \cdot \vec{r_0} = 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 = 0$$

$$(a) \vec{r_0} \cdot \vec{r_0} = 0$$

$$(ii) (\vec{r_0} \cdot \vec{r_0}) = 2(\vec{r_0} \cdot \vec{r_0}) = 2 \cdot (2 \cdot 2)$$