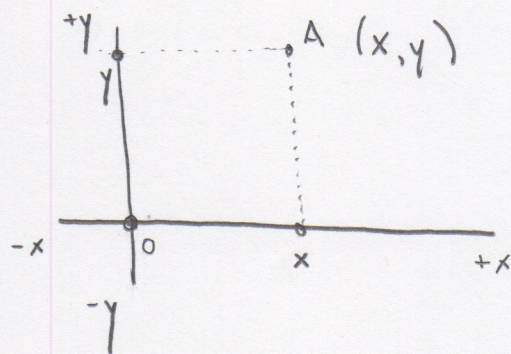
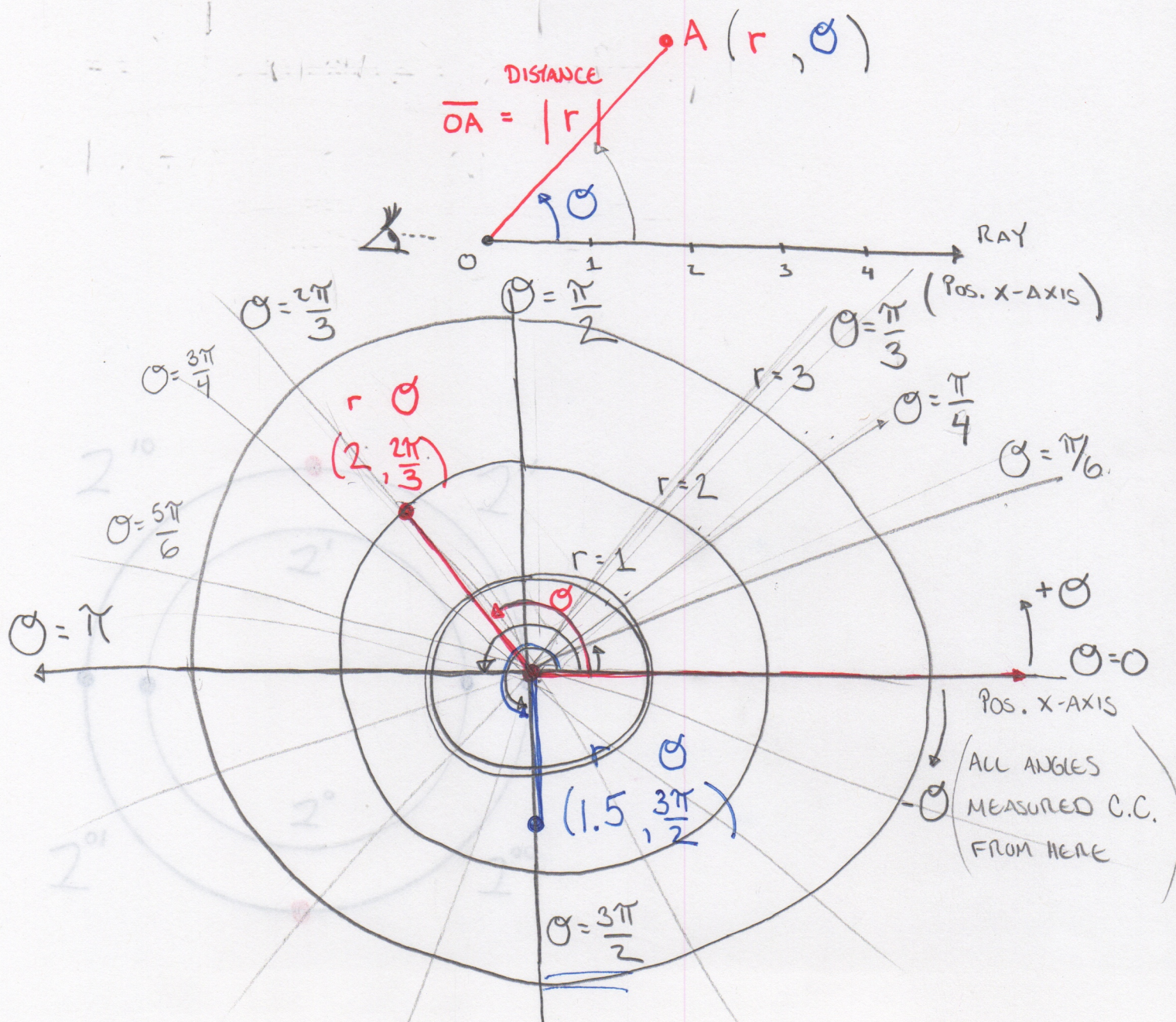


§ 11.3 Polar coordinates.

CARTESIAN COORD. (x, y)

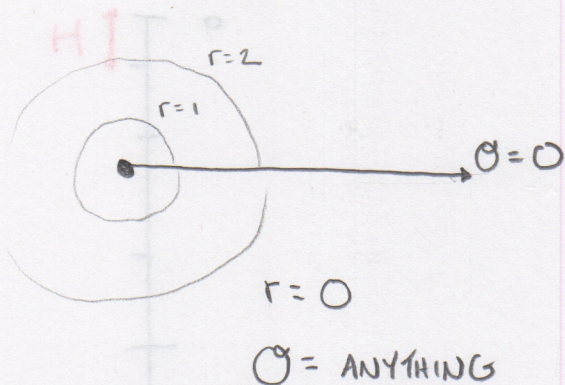


NEW COORDINATE SYSTEM (r, θ)



POLAR COORD. ARE NOT UNIQUE :

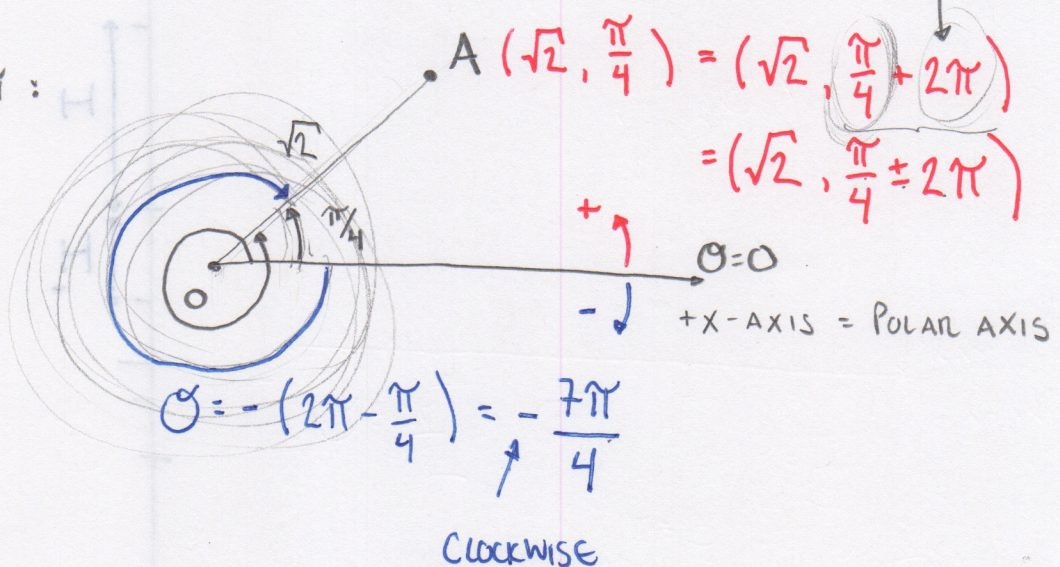
ORIGIN :



$(0,0)$
 $(0,\pi)$
 $(0,\frac{11\pi}{6})$
 $\vdots$

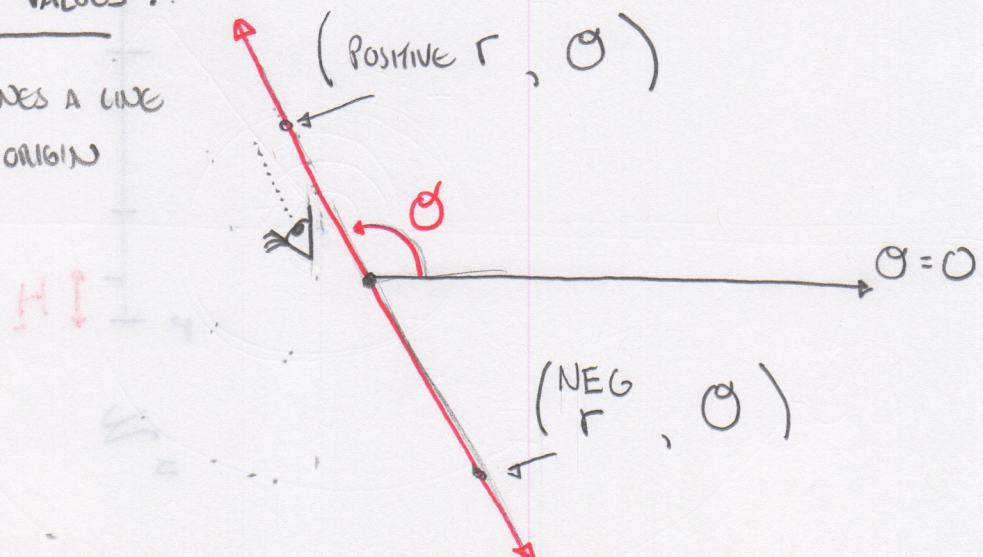
COMPLETE
REVOLUTION

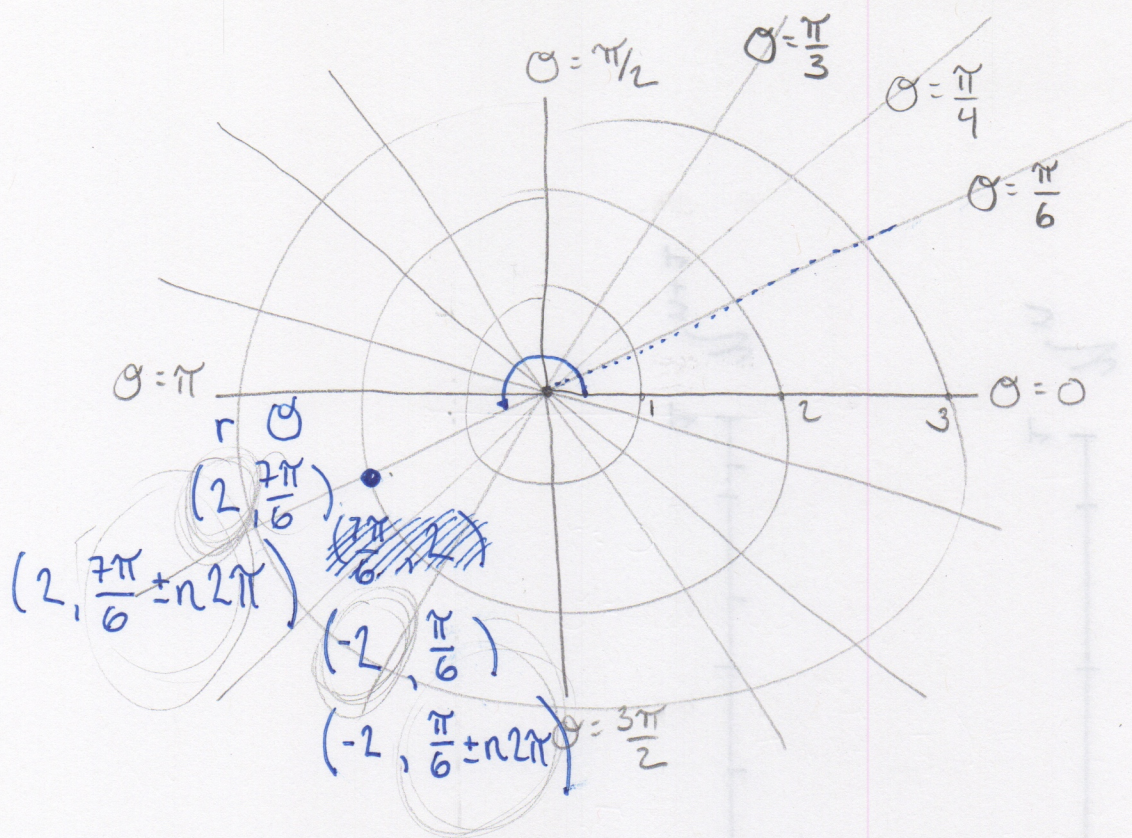
ANY POINT :



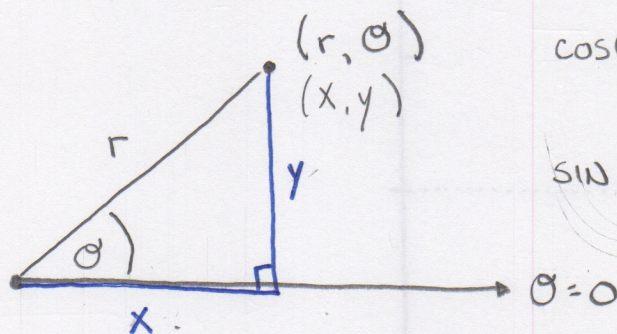
NEGATIVE r VALUES :

θ DETERMINES A LINE
THROUGH ORIGIN





CONVERTING FROM POLAR TO CARTESIAN COORD.



$$\cos \theta = \frac{x}{r} \Rightarrow$$

$$x = r \cos \theta$$

$$\sin \theta = \frac{y}{r} \Rightarrow$$

$$y = r \sin \theta$$

CONVERTING FROM CARTESIAN TO POLAR COORD.

PYTH. THM. :

$$x^2 + y^2 = r^2$$

$$\rightarrow$$

$$r = \pm \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}$$

$$\rightarrow$$

$$\theta = \tan^{-1} \frac{y}{x} < \frac{\pi}{2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) \pm n\pi$$

GRAPHS OF POLAR EQUATIONS:

SET OF ALL POINTS (r, θ)
THAT SATISFY THE EQUATION.

$$r = 1 + 2r \cos \theta$$

REWRITE WITH x, y :

$$r^2 = (1 + 2r \cos \theta)^2$$

$$x^2 + y^2 = (1 + 2[x])^2$$

$$x^2 + y^2 = 1 + 4x + 4x^2$$

$$-1 = 3x^2 + 4x - y^2 \quad (\text{circle})$$

$$-1 = 3\left(x^2 + \frac{4}{3}x\right) - y^2$$

$$-1 + \frac{4}{3} = 3\left(x + \frac{2}{3}\right)^2 - y^2$$

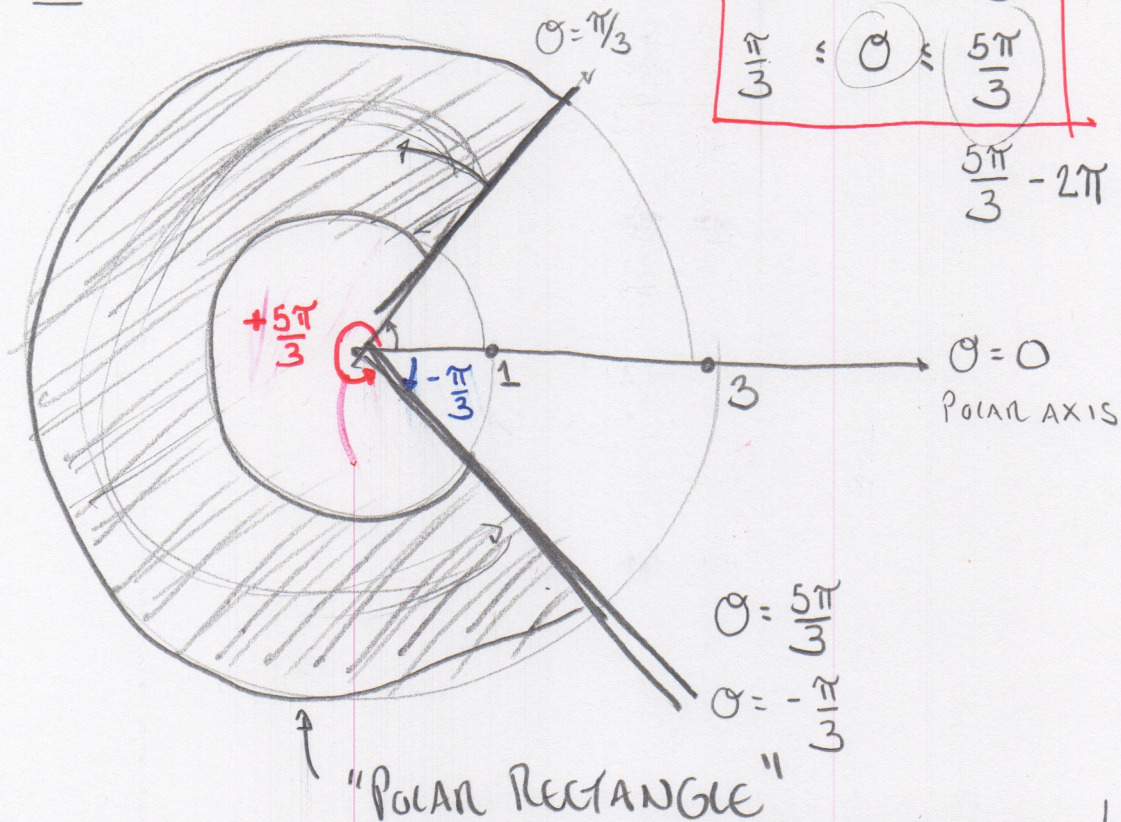
$$\frac{1}{3} = 3\left(x + \frac{2}{3}\right)^2 - y^2 \quad (\text{ELLIPSE})$$

ex:

SKETCH THE POLAR REGION

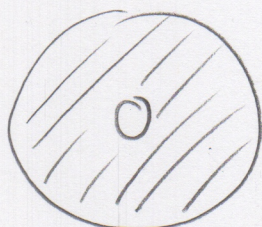
$$\boxed{\begin{matrix} 1 \leq r \leq 3 \\ \frac{\pi}{3} \leq \theta \leq \frac{5\pi}{3} \end{matrix}}$$

$$\frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}$$

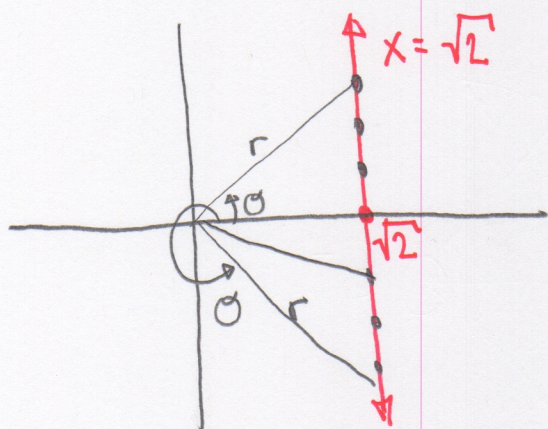


PIECES OF DISKS / ANNULI

ANNULUS :



GIVE POLAR EQ FOR THE VERTICAL LINE $x = \sqrt{2}$



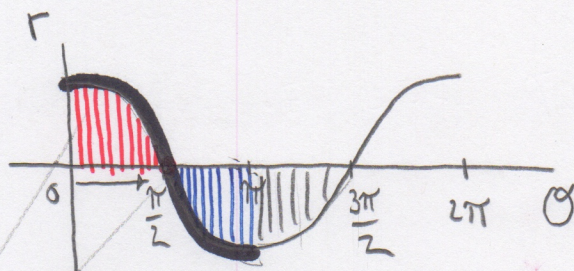
$$\downarrow$$

$$r \cos \theta = \sqrt{2}$$

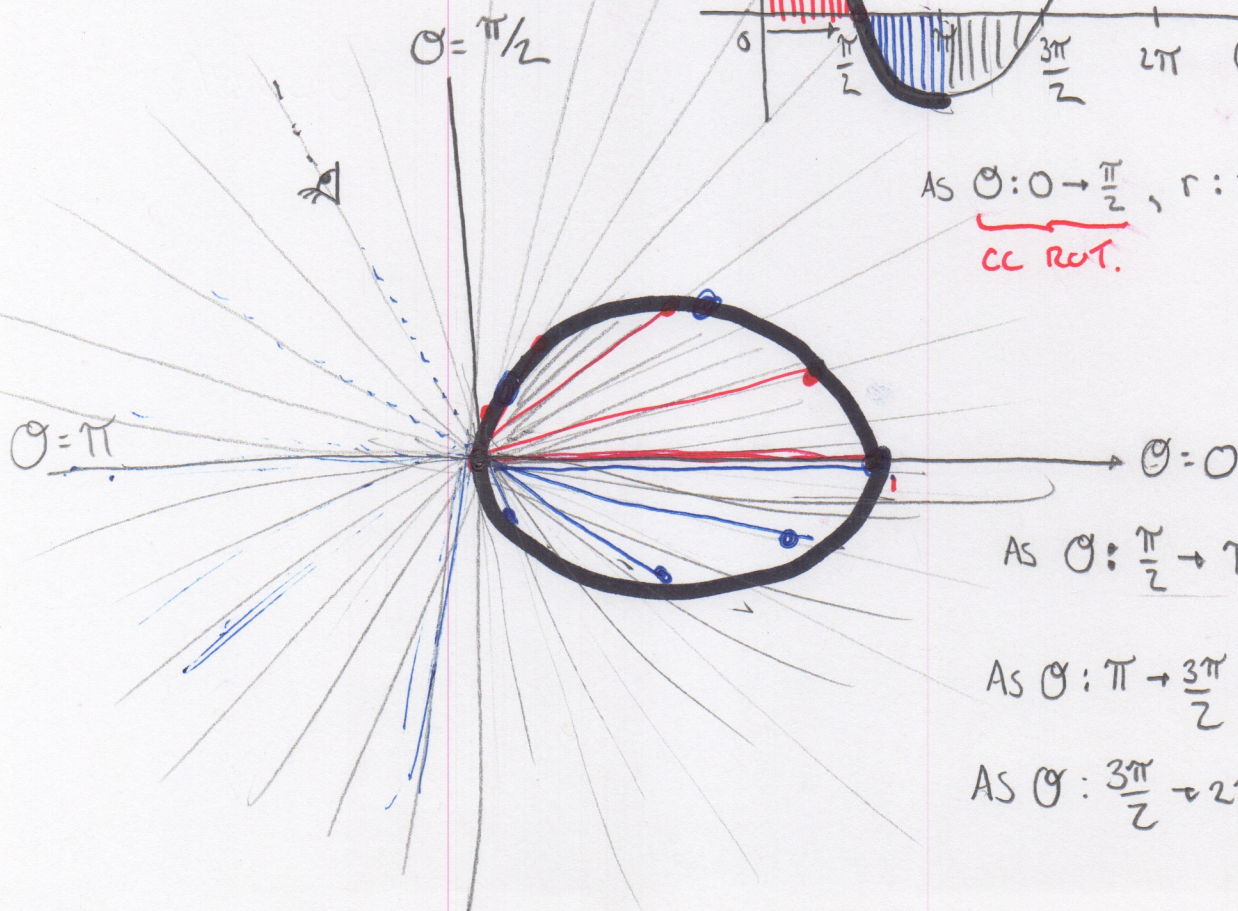
$$r = \frac{\sqrt{2}}{\cos \theta} \rightarrow \boxed{r = \sqrt{2} \sec \theta}$$

SKETCH THE POLAR GRAPH $r = \cos \theta$

CHEAT: CARTESIAN GRAPH:



AS $\theta: 0 \rightarrow \frac{\pi}{2}$, $r: 1 \rightarrow 0$
CC ROT.



AS $\theta: \frac{\pi}{2} \rightarrow \pi$, $r: 0 \rightarrow -1$

AS $\theta: \pi \rightarrow \frac{3\pi}{2}$, $r: -1 \rightarrow 0$

AS $\theta: \frac{3\pi}{2} \rightarrow 2\pi$, $r: 0 \rightarrow 1$

GRAPHING Polar EQ'S

$$y = f(x)$$

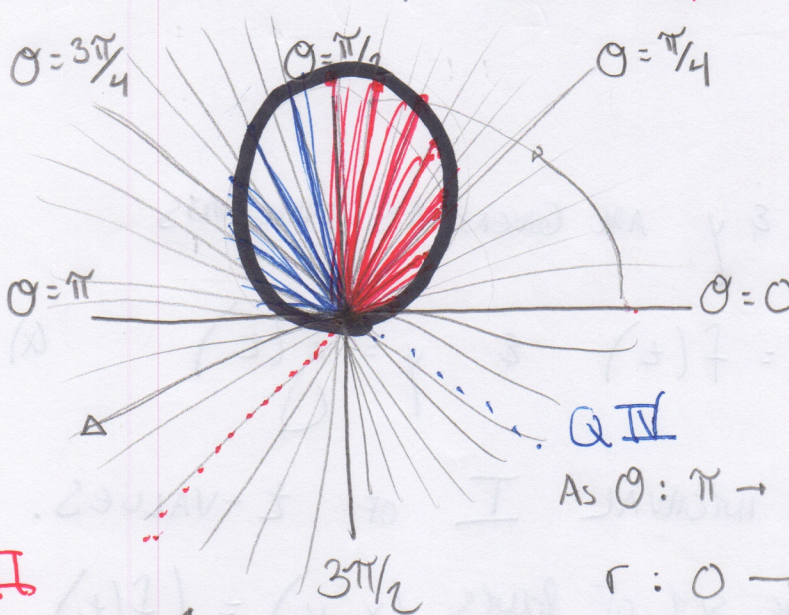
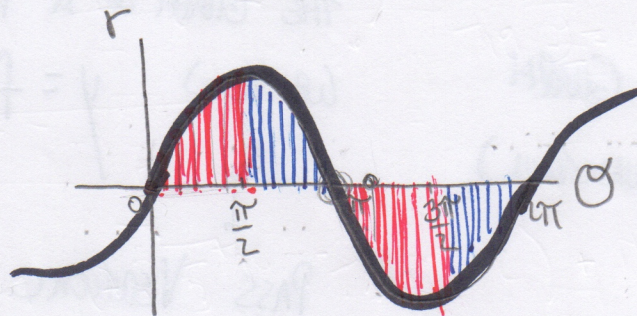
Polar EQ: $r = f(\theta)$

Dep. Var.

Ind. Var.

$$r = \sin \theta$$

1st GRAPH CARTESIAN:



AS $\theta: 0 \rightarrow \pi/2$

WE ROTATE C.C.

AS $\theta: \pi/2 \rightarrow \pi$

WE ROTATE CC

r SHRINKS FROM 1 TO 0

Q IV

AS $\theta: \pi \rightarrow 3\pi/2$ WE ROTATE CC

$r: 0 \rightarrow -1$ (BACKWARDS)

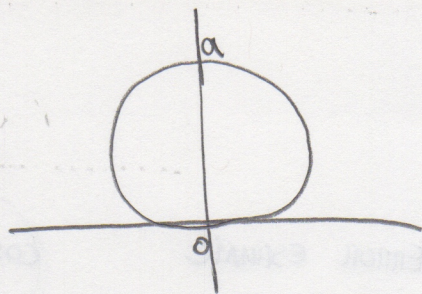
AS $\theta: 3\pi/2 \rightarrow 2\pi$

$r: -1 \rightarrow 0$

Q III

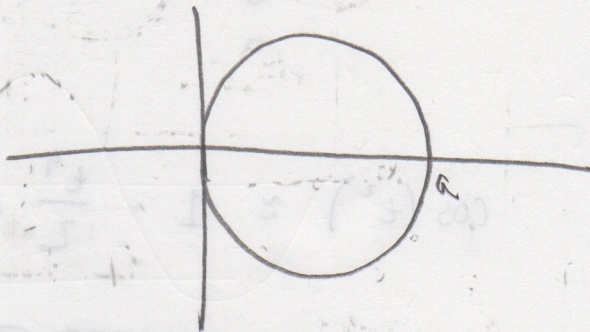
$$r = \sin \theta$$

Note: Polar Curve $r = a \sin \theta$



IS A CIRCLE WITH DIAMETER a
THROUGH ORIGIN & $(0,a)$ cent.

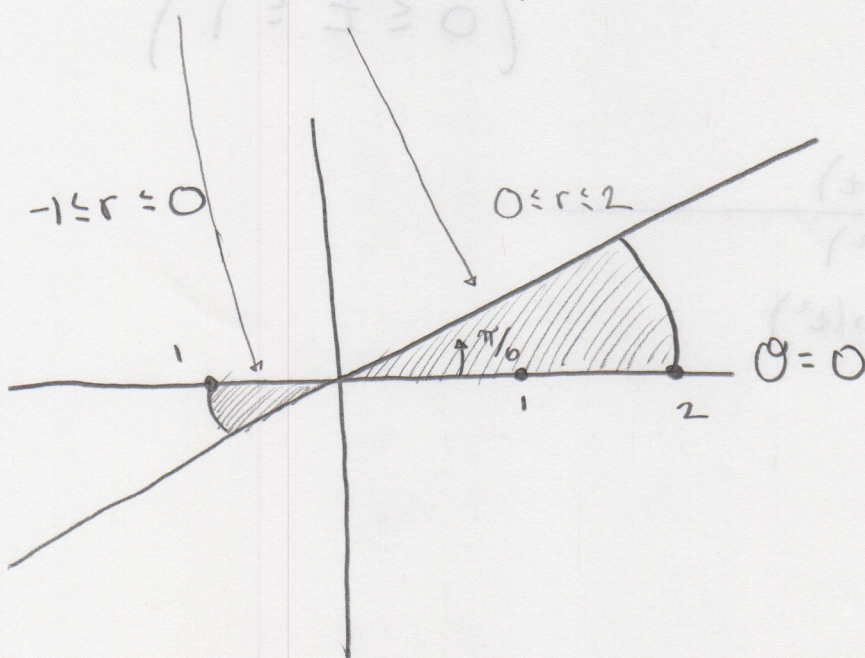
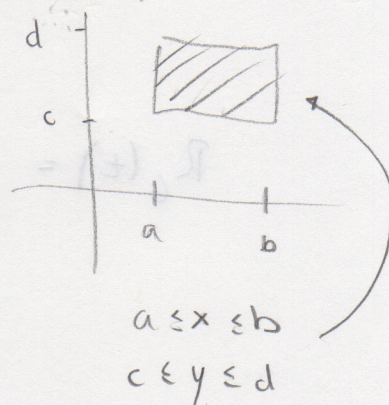
$$r = a \cos \theta$$



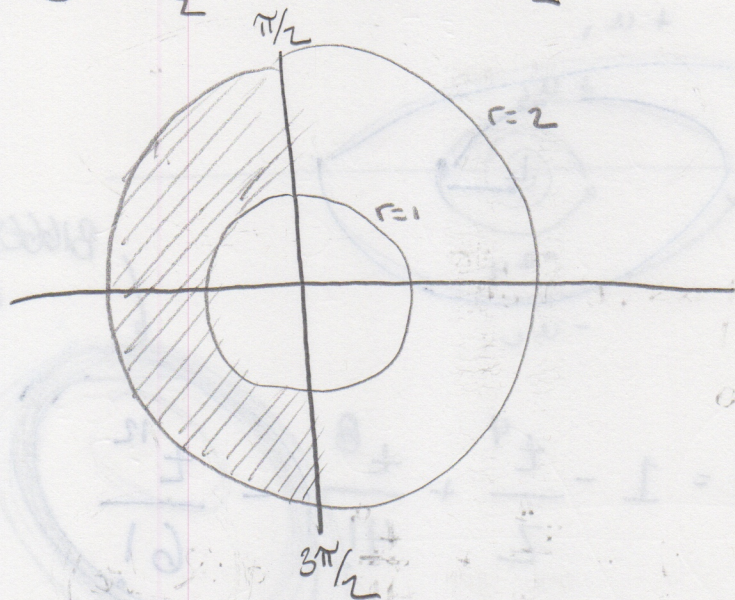
CIRCLE WITH
DIAMETER a ,
THROUGH ORIGIN &
 $(a,0)$ cent.

ex. SKETCH THE PAIR REGION

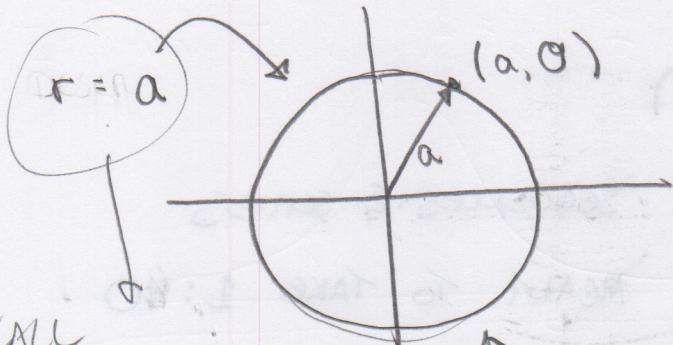
$$0 \leq \theta \leq \pi/6, \quad -1 \leq r \leq 2$$



$$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \quad 1 \leq r \leq 2$$



ex.

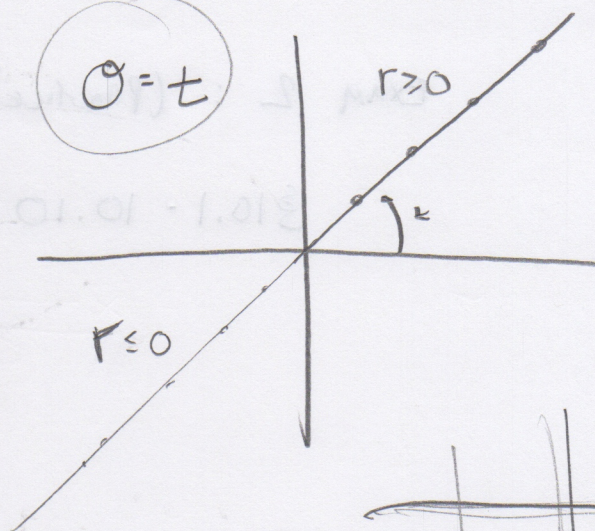


RECALL

$$r^2 = x^2 + y^2$$

$$\begin{aligned} a^2 &= r^2 \\ a^2 &= x^2 + y^2 \end{aligned}$$

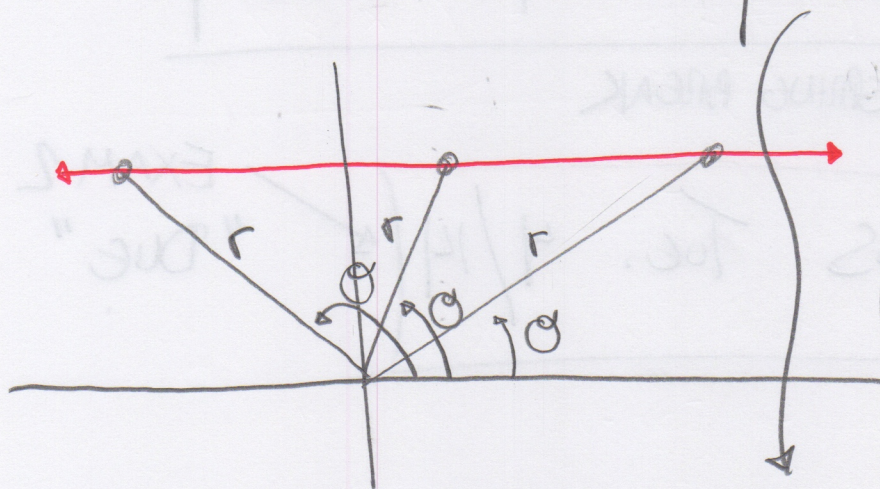
$$\theta = t$$



$$\begin{aligned} x &= c \\ y &= c \end{aligned}$$

ex.

GIVE A POLAR EQ FOR THE HORIZONTAL LINE WITH CARTESIAN EQ. $y = 5$



RECALL:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r \sin \theta = 5$$

$$r = \frac{5}{\sin \theta}$$

$$\rightarrow \boxed{r = 5 \csc \theta}$$