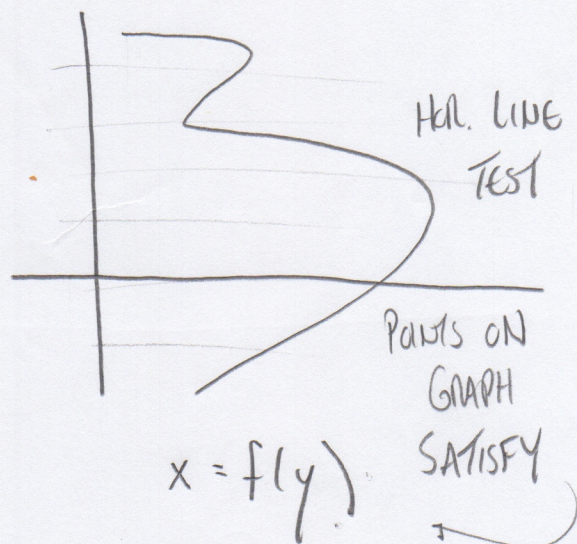
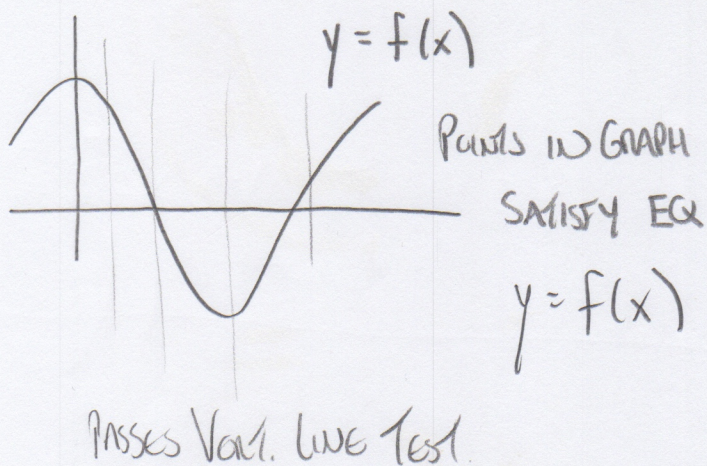


§ 11.1 PARAMETRIZATIONS OF PLANE CURVES

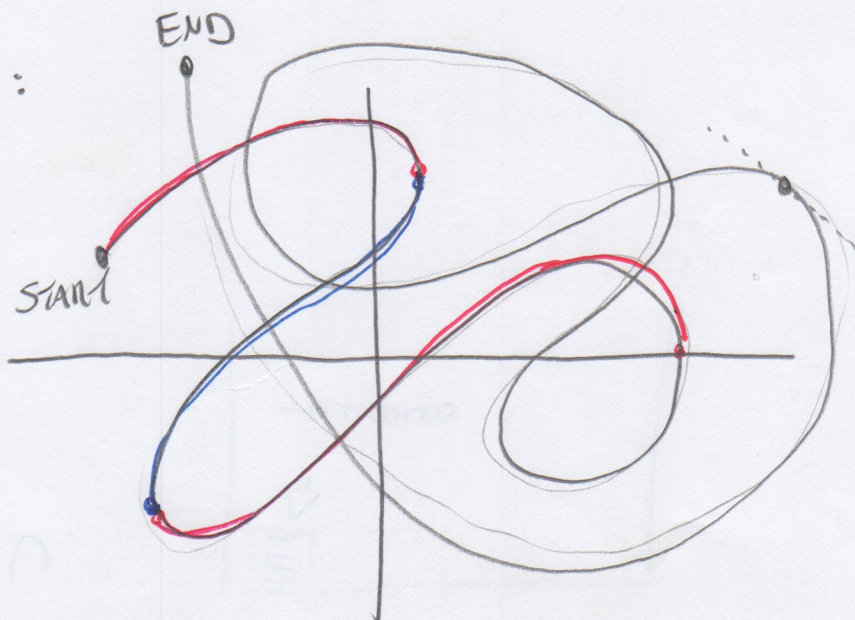
Def: A GRAPH IN THE xy -PLANE IS A COLLECTION OF POINTS (x, y) .

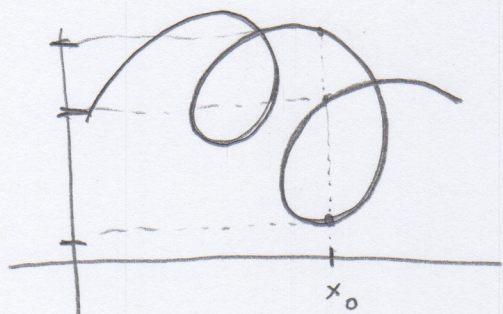


SOME GRAPHS ARE GRAPHS OF FUNCTIONS



BUT NOT ALL:





$$y = f(x_0) = ?$$

INTRODUCE A THIRD VARIABLE t .

Let both x & y depend on t
(Parameter)

Def: IF x & y ARE GIVEN AS FUNCTIONS

$$(*) \quad x = f(t), \quad y = g(t)$$

OVER AN INTERVAL I OF t -VALUES,

THEN THE SET OF POINTS $(x, y) = (f(t), g(t))$

DEFINED BY THESE EQUATIONS IS A

PARAMETRIC CURVE. THE EQUATIONS $(*)$

ARE CALLED PARAMETRIC EQUATIONS FOR THE CURVE.

t IS CALLED THE PARAMETER.

TO EVERY VALUE
FOR t THERE IS
A CORRESPONDING POINT
ON THE GRAPH

e.g.

$$x = 4 + 3t$$

$$y = t^2 - 3t + 2$$

from $-1 \leq t \leq 4$ END

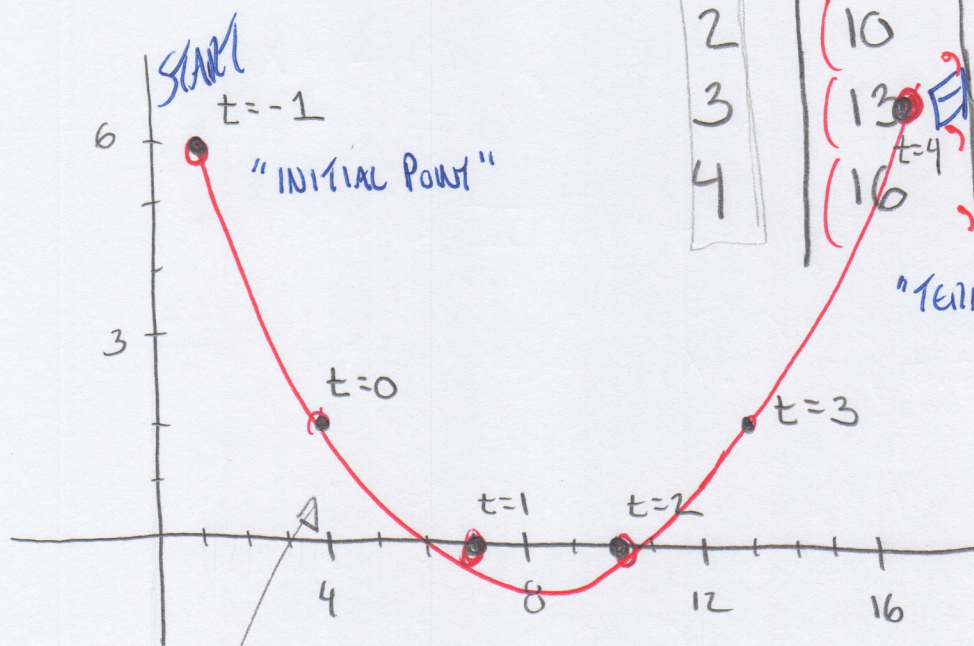
PARAMETRIC EQ'S.

$= f(t)$ ← x-coord of particle at time t

$= g(t)$ ← y-coord of particle at time t

PARAMETER

t	x	y
-1	(1, 6)	
0	(4, 2)	
1	(7, 0)	
2	(10, 0)	
3	(13, 2)	
4	(16, 6)	



$$x = 4 + 3t$$

AS t INCREASES,

x INCREASES → PARTICLE MOVES TO THE RIGHT.

SHOW THAT THIS PARAMETRIC CURVE IS A PARABOLA.

$$\begin{array}{ccc} \text{Circled Equations} & \xrightarrow[t]{\text{Solve For}} & \text{Circled Equation} \\ \begin{array}{l} x = 4 + 3t \\ y = t^2 - 3t + 2 \end{array} & & t = \frac{x-4}{3} \end{array}$$

$$y = \left(\frac{x-4}{3} \right)^2 - 3 \left(\frac{x-4}{3} \right) + 2 = ax^2 + bx + c$$

Parameter has been removed!

y is a parabola ✓

CAN'T ALWAYS DO THIS.

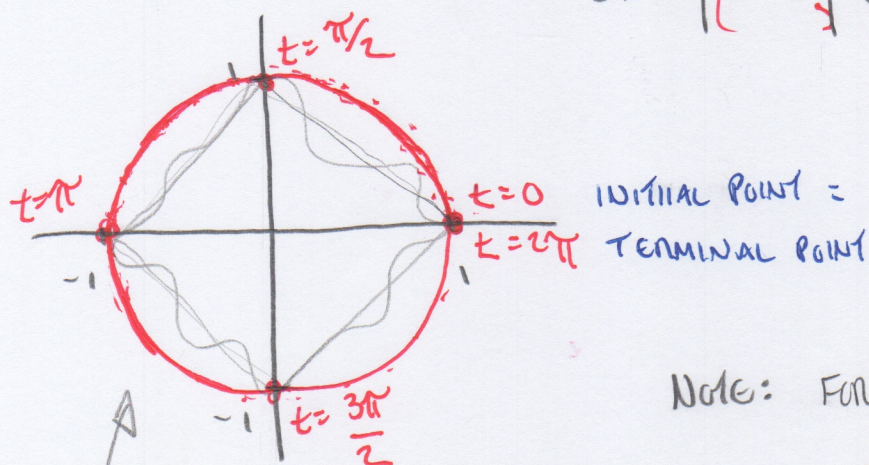
e.g.

$$x = \cos t$$

$$y = \sin t$$

$$0 \leq t \leq 2\pi$$

t	x	y
0	1	0
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1
2π	1	0



Note: For all $0 \leq t \leq 2\pi$

$$\cos^2 t + \sin^2 t = 1$$

$$x^2 + y^2 = 1$$

UNIT CIRCLE

ALL x, y ARE
DISTANCE 1 FROM
(0,0)

PARAMETRIC CURVE IS UNIT CIRCLE

DRAWN ONCE, COUNTER CLOCKWISE,
STARTING & ENDING AT (1,0).

eg. $x = \sqrt{t} \rightarrow f(t)$
 $y = t \rightarrow g(t)$

$t \geq 0$

start
 $0 \leq t < \infty$ No End

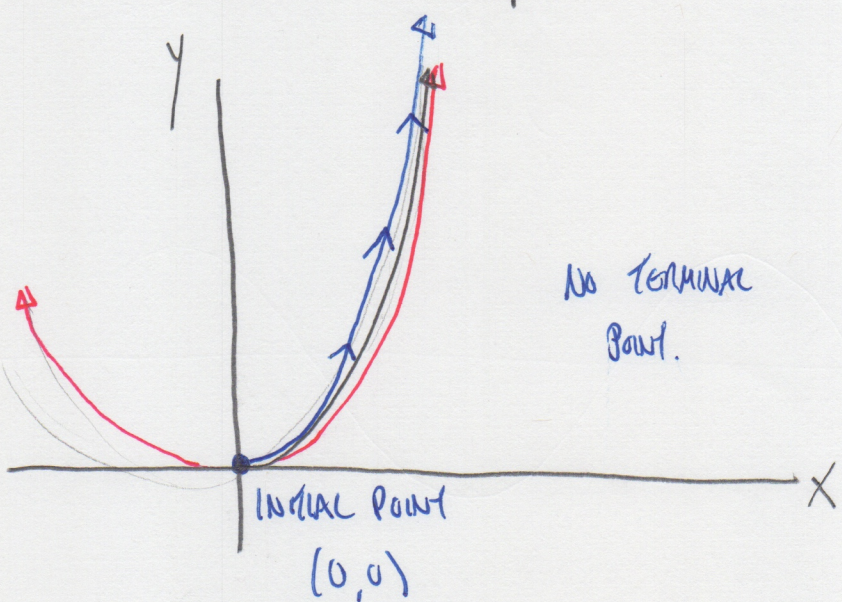
UNLESS OTHERWISE SPECIFIED,
 DOMAIN IS SET OF ALL VALUES FOR t
 S.T. BOTH f & g ARE DEFINED.

REMOVE PARAMETERIZATION: $y = t$

$x = \sqrt{t} = \sqrt{y}$

$x = \sqrt{y}$

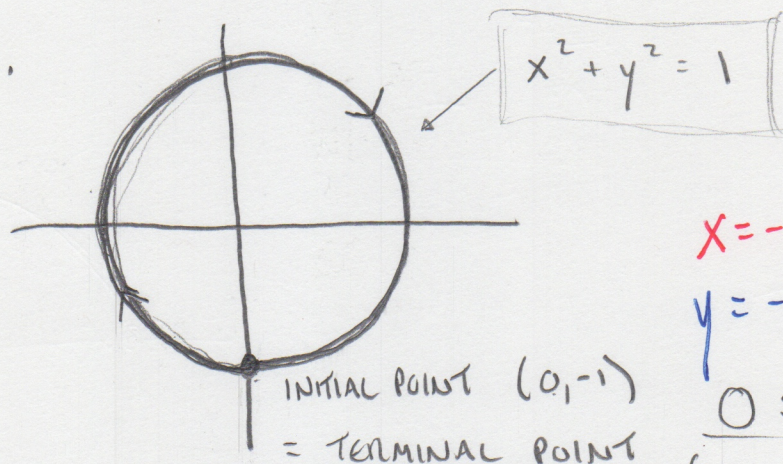
$x^2 = y$



PARAMETERIZED CURVE LIES (LIES) ON
 THIS CURVE.

BUT THIS CURVE INCLUDES MORE POINTS THAN
 PARAM. CURVE.

eg. Give Parametric Equations for a particle that moves
 along the unit circle with center $(0,0)$,
 twice clockwise, beginning and ending at
 $(0, -1)$.



$$x = -\sin t$$

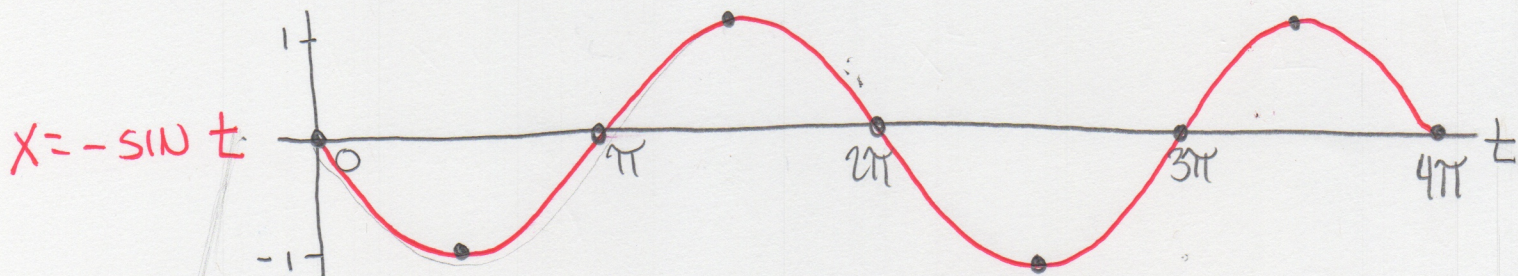
$$y = -\cos t$$

$$0 \leq t \leq 4\pi$$

$$(12\pi \leq t \leq 16\pi)$$

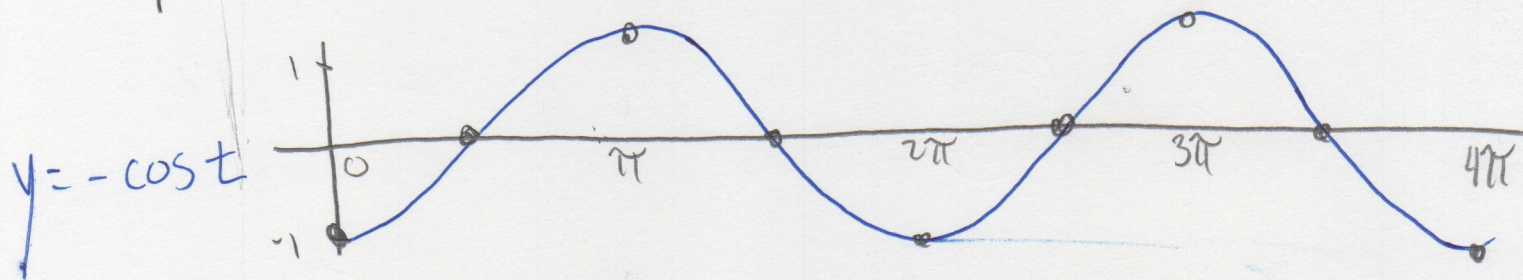
START

x : $0 \searrow -1 \nearrow 0 \nearrow 1 \searrow 0 \searrow -1 \nearrow 0 \nearrow 1 \searrow 0$



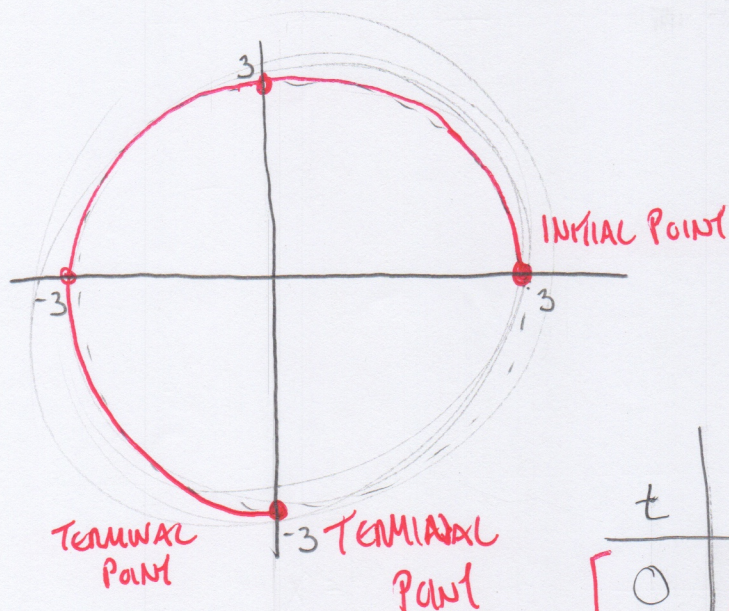
START

y : $-1 \nearrow 0 \nearrow 1 \searrow 0 \searrow -1 \nearrow 0 \nearrow 1 \searrow 0 \searrow -1$



e.g.

GIVE PARAMETRIC EQ'S TO DESCRIBE THE POSITION
OF A PARTICLE THAT MOVES ALONG A CIRCLE
WITH RADIUS 3, CENTERED AT $(0,0)$,
AND MOVES COUNTER CLOCKWISE FROM
 $(3,0)$ TO $(0,-3)$



$$\begin{aligned} x &= 3 \cos t \\ y &= 3 \sin t \end{aligned} \quad \begin{aligned} &\downarrow \\ 0 &\leq t \leq \frac{3\pi}{2} \end{aligned}$$

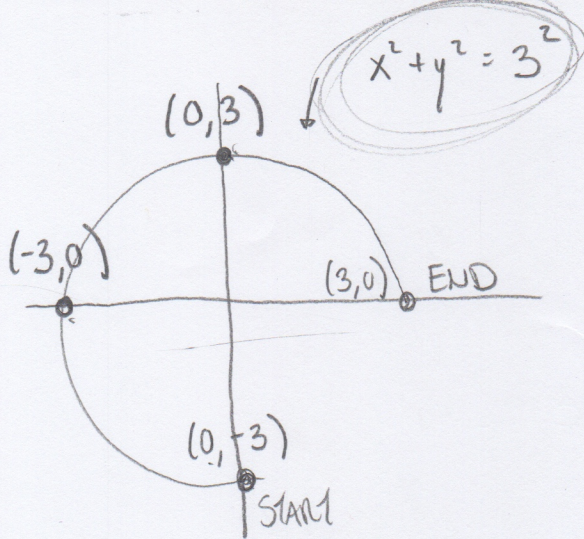
$$x^2 + y^2 = 9(\cos^2 t + \sin^2 t) = 3^2$$

I

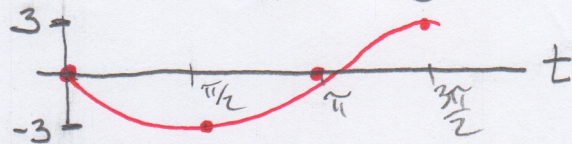
t	x	y
0	3	0
$\frac{\pi}{2}$	0	3
π	-3	0
$\frac{3\pi}{2}$	0	-3

TERMINAL POINT

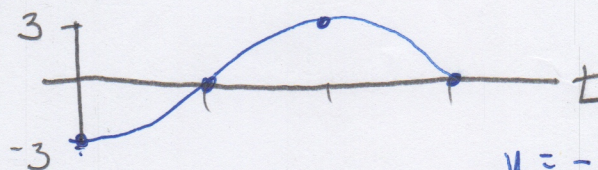
FOLLOW-UP: START AT $(0,-3)$, MOVE CLOCKWISE TO $(3,0)$
ALONG SAME CIRCLE.



$x: 0 \searrow -3 \nearrow 0 \nearrow 3$ $x = -3 \sin t$



$y: -3 \nearrow 0 \nearrow 3 \searrow 0$



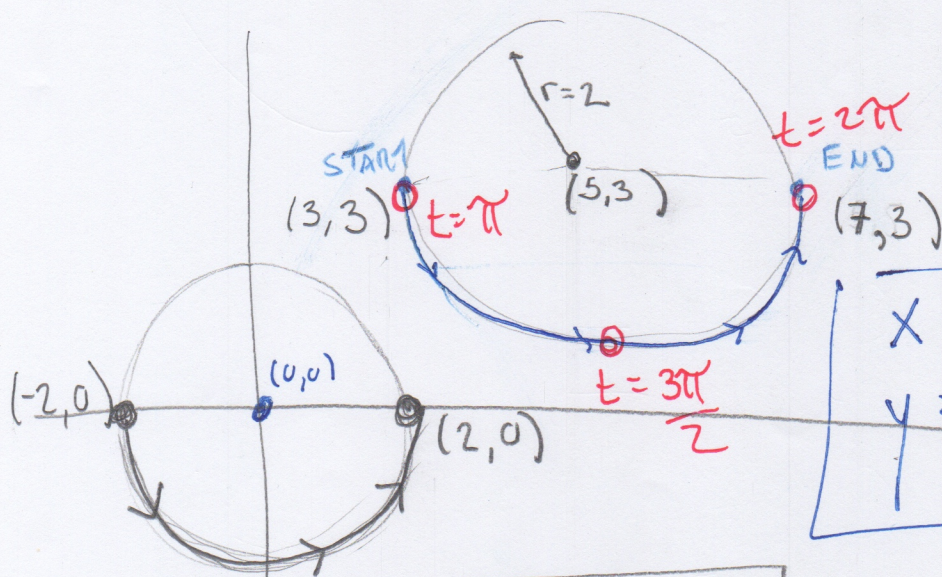
$y = -3 \cos t$

PARAMETRIC EQ's :

$x = -3 \sin t$

$0 \leq t \leq \frac{3\pi}{2}$

$y = -3 \cos t$



$-\pi \leq t \leq 0$

$\pi \leq t \leq 2\pi$

$x = 5 + 2 \cos t$

$y = 3 + 2 \sin t$

$3\pi \leq t \leq 4\pi$

$x = 2 \cos t$

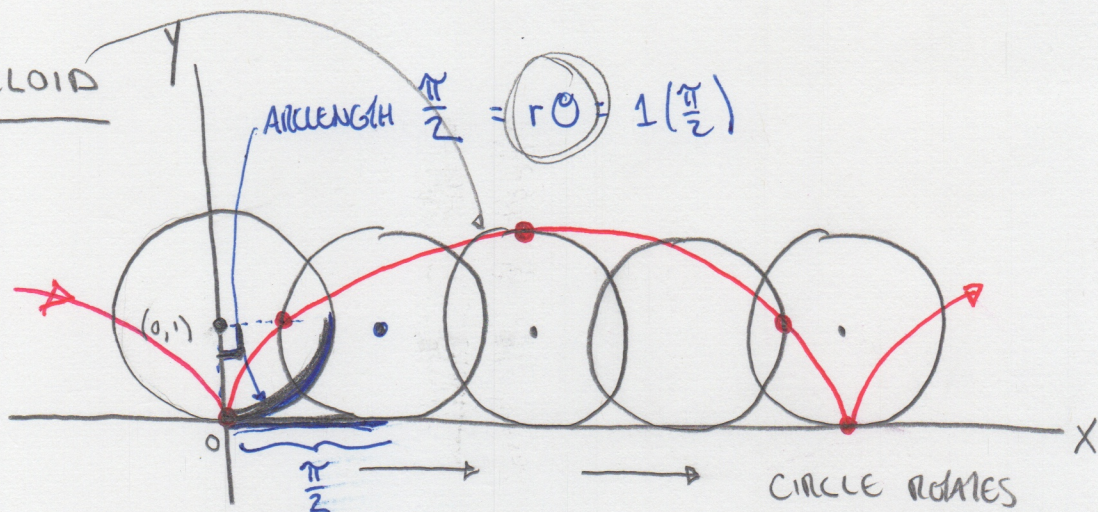
$y = 2 \sin t$

$\pi \leq t \leq 2\pi$

t	x	y
π	3	3
$3\pi/2$	5	1
2π	7	3

ex.

CYCLOID

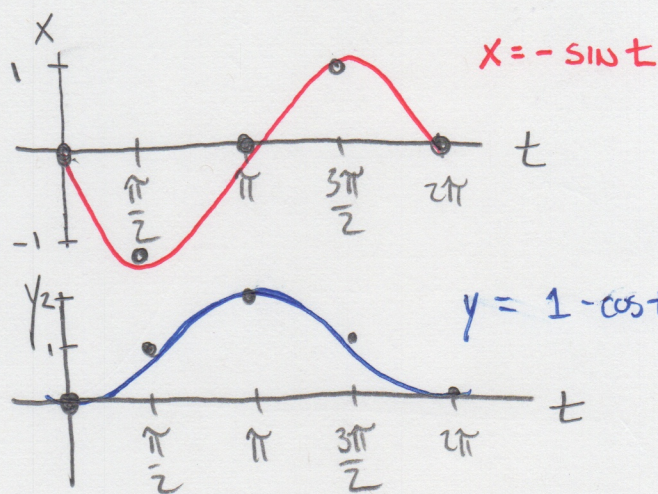
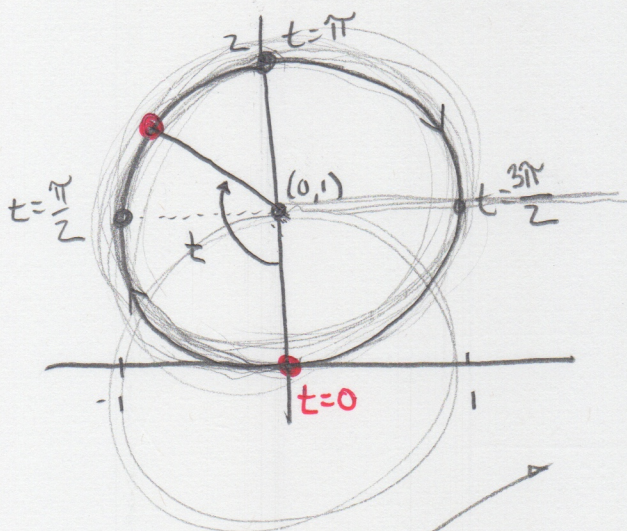


GIVE PARAMETRIC EQ'S FOR THIS CYCLOID.

CIRCLE ROLATES
CLOCKWISE.

(1) CIRCLE SPINS IN PLACE:

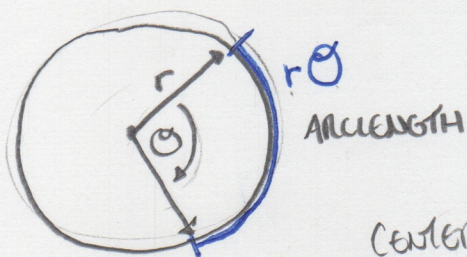
Let t = RADIANS THAT CIRCLE
HAS ROLATED.



(2)

POSITION WHEN CENTER IS STATIONARY.

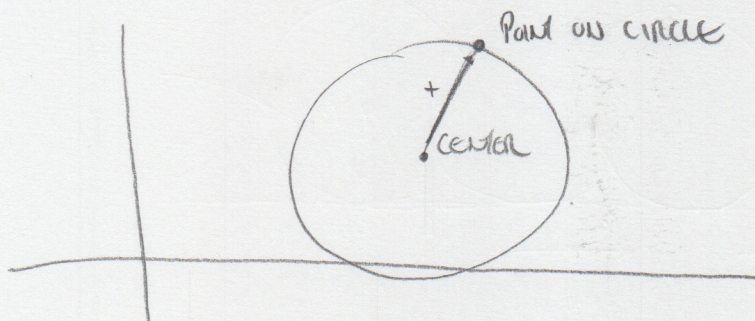
WHEN ROLLING, THE CENTER MOVES TO THE RIGHT.



DISTANCE CIRCLE HAS ROLLED
= $r t = t$.

CENTER OF CIRCLE IS MOVING TO THE RIGHT
↓ $y = 1$, $x = t$.

Center: $y = 1, x = t$



$$x = t + (-\sin t)$$

$$y = 1 - \cos(t)$$

SINGLE CYCLOID

$$0 \leq t \leq 2\pi$$

