

Please put away all papers and electronic devices. Show enough work that it is clear how you arrived at your answer. Put a box/circle around your final answer to each question. Good luck!

1. Consider the following function of x .

$$F(x) = \int_1^x \frac{1}{t} dt$$

Hint: there is a common name for this function.

- (a) (6 points) Evaluate the derivative $\frac{d}{dx} F(x)$.

- (b) (6 points) Evaluate the integral $\int_1^e F(x) dx$.

2. (6 points) Use the definitions of $\sinh x$ and $\cosh x$ to simplify the following expression.

$$\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$$

3. Evaluate the integrals.

(a) (8 points) $\int \sec^3 x \tan^5 x \, dx$

(b) (8 points) $\int (x^2 + 1)e^{2x} \, dx$

4. Evaluate the integrals.

(a) (8 points) $\int e^{-x} \sin(\pi x) \, dx$

(b) (8 points) $\int_0^1 \sin^{-1} x \, dx$

5. Evaluate the integrals.

(a) (8 points) $\int \frac{1}{x^2\sqrt{x^2+1}} dx$

(b) (8 points) $\int \frac{x+3}{2x^3+8x} dx$

6. (6 points) Give the *form* of the partial fraction decomposition for the following rational function. Leave your answer in terms of constant coefficients A, B, C , etc., which you *do not* need to solve for.

$$\frac{3x^5 - 2x + 1}{(x - 1)^3(x^2 + 3)^2}$$

7. Consider the following integral

$$\int_0^2 \frac{x + 1}{\sqrt{4 - x^2}} dx$$

- (a) (4 points) Briefly explain why the integral is improper.

- (b) (8 points) The integral converges. What does it converge to?

8. Use an appropriate test for convergence to decide whether each of the following improper integrals converges or diverges.

(a) (8 points) $\int_2^{\infty} \frac{1}{\sqrt{x} - e^{-x}} dx$

(b) (8 points) $\int_2^{\infty} \frac{1}{x\sqrt{x} - 1} dx$