Name ______________________________

Do all problems.

1. (24 points) Evaluate the following integrals
   (a) $\int_0^{\frac{\pi}{2}} t^2 \cos t \mathrm{d}t$
   (b) $\int \sec^3 t \tan^3 t \mathrm{d}t$.
   (c) $\int_0^1 \arcsin x \mathrm{d}x$
   (d) $\int \frac{t^3-1}{t^3+t} \mathrm{d}t$.

2. (15 points) State for each series whether it converges absolutely, converges conditionally, or diverges. Name a test which supports your conclusion and justify why it applies, by showing a calculation or giving an explanation.
   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{3n^2+n+1}$.
   (b) $\sum_{n=2}^{\infty} \frac{2^n+5}{3^n}$.
   (c) $\sum_{n=1}^{\infty} \frac{\cos(n^2)}{n\sqrt{n}}$

3. (8 points) (a) Show that the limit $\lim_{(x,y)\to(0,0)} \frac{x^2 \cos(x+y)}{x^2+y^2}$ does not exist.
   (b) Without attempting to evaluate it, determine whether the integral $\int_1^{\infty} \frac{1}{\sqrt{x^5+2}} \mathrm{d}x$ converges. Explain your reasoning.

4. (8 points) (a) Using a known series express $F(x) = \int_0^x \sin(t^2) \mathrm{d}t$ as a power series centered at $x = 0$. Find the series' radius of convergence. Justify your answer.
   (b) Use your power series in (a) to estimate $\int_0^1 \sin(t^2) \mathrm{d}t$ with an error less than .01. Justify your answer has the desired accuracy.

5. (4 points) For what values of x does the series

$$1 - \frac{1}{2}(x-3) + \frac{1}{2^2}(x-3)^2 + \ldots + (-\frac{1}{2})^n (x-3)^n + \ldots$$

   converge? What is its sum when $x = \frac{5}{2}$?

6. (6 points) Use the Trapezoid rule to estimate $\int_{-1}^{1} (x^2+1) \mathrm{d}x$ with $n = 4$. Compare your estimate with the exact value of the integral.

7. (12 points) (a) Compute an equation for the plane which contains the point $(1, 0, 1)$ and the line given parametrically by the equations $x = 2t,\ y = 2 + t,\ z = 2 - t$.
   (b) How much work is required to slide a crate 25 m along a loading dock by pulling on it with a 220−N force at an angle of 30° from the horizontal.
   (c) If $\vec{u}_1$ and $\vec{u}_2$ are orthogonal unit vectors, and $\vec{v} = -\frac{2}{3}\vec{u}_1 - \frac{1}{5}\vec{u}_2$, find $\vec{v} \cdot \vec{u}_2$.

8. (8 points) Let $f(x, y) = 4 - x^2 - y^2$.
   (a) Sketch the surface $z = f(x, y)$.
   (b) Sketch the level curves $f(x, y) = c$ for $c = -2, 0, 3, 4$.

9. (8 points) Find the area of the triangle with vertices $P = (1, -1, 0), Q = (2, 1, -1)$, and $R = (-1, 1, 2)$. Find a unit vector perpendicular to the plane through the points P, Q, and R.

10. (8 points) Evaluate $\frac{\mathrm{d}w}{\mathrm{d}t}$ for each of the following.
    (a) $w = 2^{\tan^{-1}(2t)}$
    (b) $w = \ln\left(\sqrt{x^2 + z}\right), x = t^2, z = 3t$.