Name:_____ **11/1/2018**
Math 20300-ST Calculus III **Midterm Exam**

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Let P be the point $(3, -1, 2)$ And let C be the circle with equation

$$(x-1)^2 + (y-2)^2 + (z+4)^2 = 9. \tag{C}$$

   (a) (4 points) What is the distance between P and the center of C?

   (b) (4 points) What is the distance between P and C?

2. (8 points) Two intersecting lines l_1 and l_2 have the following parametric equations.

$$x = 6 - 4s, \quad y = 9 - 8s, \quad x = -4 + s \tag{l_1}$$

$$x = 2 + 4t, \quad y = 1 + 7t, \quad x = -3 - 4t \tag{l_2}$$

   Find the angle between the lines at the point where they intersect.
   Note: You do not need to find where the lines intersect and your answer may be left as a trig/inverse-trig expression.

3. (a) (8 points) Give a vector equation or parametric equations for the line through the point $(3, 1, -4)$ that is parallel to both of the following planes.

$$x - y - z = 7 \qquad (P_1)$$

$$x + y - 2z = -1 \qquad (P_2)$$

(b) (4 points) Find the point at which the line from part (a) intersects the yz-plane.

4. (8 points) Give a vector equation or parametric equations for the tangent line to the curve

$$\vec{r}(t) = \langle\, e^{-t}\cos t,\ e^{-t}\sin t,\ e^{-t} \,\rangle \qquad (\vec{r})$$

at the point $(1, 0, 1)$.

5. (4 points) Sketch the domain of the function $f(x, y) = \dfrac{\ln(x^2 + y^2 - 4)}{(x - 3)(y - 4)}$.

6. (8 points) Evaluate the limit or show it does not exist. Justify your answer.

$$\lim_{(x,y)\to(0,0)} \frac{2x \sin y}{x^2 + y^2}$$

7. Let $f(x, y) = \ln(2x + y)$.

(a) (4 points) Find all second partial derivatives at f.

(b) (8 points) Find an equation for the tangent plane to the surface $z = f(x, y)$ at the point $(-1, 3, 0)$.

8. At points (x, y, z) in a region of space for which $x^2 + y^2 \geq 1$ and $z \geq 0$ there is an electric charge

$$E(x, y, z) = z + z \ln(x^2 + y^2). \qquad (E)$$

(a) (8 points) Find the rate at which the electric charge is changing at $P(1, 0, 2)$ in the direction towards the point $Q(4, 4, 7)$.

(b) (4 points) Find the direction of greatest increase in E at the point $P(1, 0, 2)$.

(c) (4 points) At each point (s, t) on the ground in a physics lab, the electric charge at position

$$(x, y, z) = (s + t, s - t, 2st)$$

is measured. Find the rate $\dfrac{\partial E}{\partial s}$ at which the electric charge is changing with respect to s at the point $(s, t) = (1, 1)$.