1. (10 points) Let $f(x, y) = x \sin(x + y)$. Explain why f is differentiable at the point $(-1, 1)$. Then find an equation for the tangent plane to the graph $z = f(x, y)$ at the point $(-1, 1, 0)$.

2. (8 points) Find all local maxima, local minima, and saddle points for

$$f(x, y) = 3x^2 - 12xy + 8y^3.$$

3. (12 points) At points (x, y, z) in a region of space for which $x^2 + y^2 \geq 1$ and $z \geq 0$, there is an electric charge
$$E(x, y, z) = z + z \ln(x^2 + y^2).$$

(a) Find the direction of greatest increase in E at $(1, 0, 2)$.

(b) Find the rate at which the electric charge is changing at $(1, 0, 2)$ in the direction towards the point $(4, 4, 7)$.

(c) At each point (s, t) on the ground in a physics lab, the electric charge is measured at the corresponding point with the following coordinates.

$$x = s + t \qquad y = s - t \qquad z = 2st$$

Find the rate $\dfrac{\partial E}{\partial s}$ at which the electric charge is changing with respect to s at the point $(s, t) = (1, 1)$.

4. (10 points) A laminar region $\mathcal{R}$ lies in the first quadrant, includes the origin, and is bounded by the following curves.

$$y = 2x \qquad y = 1 \qquad y = x^3$$

The density δ of the laminar at the point (x, y) is given by $\delta(x, y) = 24x^2$. Sketch the region $\mathcal{R}$ and compute its mass.

5. (10 points) Find the volume of the solid that lies below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the xy-plane.