Name:_____ **11/6/2014**

Math 203 Calculus III **Midterm Exam**

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Suppose the position of a particle at time t is given by the vector valued function

$$\vec{r}(t) = \langle t\cos t, t\sin t, t^2\rangle, \quad -\infty < t < \infty.$$

(a) (4 points) Find two different times t_1 and t_2 when the particle is at the point $P(0, \frac{\pi}{2}, \frac{\pi^2}{4})$.

(b) (4 points) Show that the particle is traveling in different directions each time it passes through the point P.

2. Consider the two planes P_1 and P_2 given by the following equations.

$$x + 3z = 2y + 4 \qquad (P_1)$$

$$3y = 2z + 10 \qquad (P_1)$$

(a) (4 points) Are the planes P_1 and P_2 parallel, perpendicular, or neither? Show the calculation that supports your answer.

(b) (4 points) Give parametric equations for the line through the point $(1, 2, -1)$ that is parallel to both planes P_1 and P_2.

(c) (4 points) Find the point at which the line you found in part (b) intersects the xz-plane.

3. (4 points) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)^2}{(x^2+y^2)^2}$$

4. (4 points) Suppose

$$w = xy + yz + zx$$

and

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = r\theta$$

Find $\dfrac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \dfrac{\pi}{2}$.

5. Let $f(x, y, z) = x^2y^3 + e^{x+z} - 2y \sin z$.

(a) (4 points) Find the directional derivative of f at $P(0, 1, 0)$ in the direction toward the point $Q(3, 5, -12)$, i.e. in the direction of the line segment from P to Q.

(b) (4 points) Find an equation of the tangent plane to the level surface $f(x, y, z) = 1$ at the point $P(0, 1, 0)$.

6. (4 points) Find the linear approximation to the function $f(x, y) = \tan^{-1}(xy^2)$ at the point $(1, 1, \frac{\pi}{4})$ and use it to approximate $f(0.9, 1.1)$.

7. (4 points) Find $\dfrac{\partial z}{\partial y}$ when z is defined to be a function of x and y implicitly by the equation

$$yz + x \ln y = z^2.$$

8. (4 points) Write a poem about a math topic we've discussed in class. For example, here is a haiku about two different ways to multiply vectors in $\mathbb{R}^n$.

Dot product, scalar.
Cross product, vector. Aha!
Two different outputs.