NAME (printed) ______________________________

NAME (signed) ______________________________

MATH 203 Final Exam

December 22, 2014

Circle your section (for example, XX, Instructor, Days, Hours):

BB, Shell, M, W 9–10:40
DD, Shell, W 11–12:40
EE, Islam, M, W, 2–3:40
LL, Adamski, T, Th 9–10:40
LM, Jitsukawa, T, Th, 10–11:40
MM, Kapsack, T, F, 11–12:40
PP, Musser, T, Th, 2–3:40
RS, Bam, T, Th 4–5:40

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total	

Instructions: Complete every question in Part I (Questions 1–7), and answer THREE (3) COMPLETE questions from Part II (Questions 8–12). In the table above mark an **X** through the two questions that you omit. **Show all work.**

No calculators or other electric devices may be used. Answers are to be left in terms of $\sqrt{7}$, π, $\ln 3$, etc. when these can not be simplified. You have **2 hours and 15 minutes to complete the exam.**

PART I: Answer ALL questions in this part (10 points each)
Show all work and simplify all answers. **NO ELECTRONIC DEVICES!!!**

1. (a) Find an equation for the plane that contains the x-axis and the point $(5,2,3)$.
   (b) Find parametric equations for the line through $(1,1,1)$ and $(1,2,3)$.
   (c) Are the plane found in (a) and the line found in (b) parallel? Show work.

2. The concentration of polutant in a thin film of poluted water at point (x, y) is $p(x, y) = xe^{x+2y}$.

(a) At the point $(2, -1)$, in which direction is the change per unit travelled of the concentration $p(x, y)$ decreasing the most rapidly?

(b) An ant crawling through the film is at position $(x, y) = (t^2 + t, t^3 - 2t)$ at time t. At what rate is the concentration at the ant's position changing per unit time at time $t = 1$?

(c) What is the rate of change in the concentration per unit distance travelled at the point $(4, -2)$ travelling towards the point $(3, 3)$?

3. Find all local maxima, local minima and saddle points of the graph of

$$f(x, y) = 2x^4 - x^2 + 3y^2 + 6y.$$

4. (a) Find the volume of the region in the first octant bounded by $x = 0$, $y = 0$, $x + y = 2$, $z = x + 3$, $z = y + 5$.

(b) Find an equation of the tangent plane to the graph of $z = \ln(e^x + e^{2y-4})$ at the point on the graph for which $(x, y) = (0, 2)$.

5. Find the mass of a lamina with density $\delta(x,y) = \dfrac{1+\sqrt{x^2+y^2}}{x^2+y^2}$ which occupies the the portion of the annular ring $\{(x,y) : 1 \le x^2+y^2 \le 4\}$ in the first quadrant.

6. State, for each series, whether it converges absolutely, converges conditionally or diverges. Name a test which supports each conclusion and show the work to apply the test.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n(2^n)}{n+2^n}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2^n}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$

7. Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n \ln(n+2)}$. (Remember to check the endpoints, if applicable.)

PART II: Answer any THREE COMPLETE questions (10 points each).

8. (a) Find the volume of the region inside the sphere $x^2 + y^2 + z^2 = 3$ and above the cone $z = -\sqrt{x^2 + y^2}$.

(b) Find the following limits or show they do not exist:

(i) $\lim_{(x,y)\to(0,0)} \dfrac{\sqrt{xy}}{x^2 + 2y^2}$ (ii) $\lim_{(x,y)\to(0,0)} \dfrac{\sqrt{xy}}{4 + x^2 + 2y^2}$

9. (a) Use differentials (linear approximation) to approximate the volume of a (2.98 cm)×(3.01 cm)×(4.02 cm) rectangular box.

(b) Find an equation of the tangent plane to the graph of $\cos(x+2y+3z) = x+2y-e^{4z}$ at the point $(2,-1,0)$.

10. (a) Find the area of the portion of the surface $z = xy + 1$ which is inside the cylinder $x^2 + y^2 = 3$

(b) Graph, labelling the coordinates of any vertices: $x^2 - y^2 - 4y + z^2 = 4$.

11. Find the mass of the solid bounded by the surfaces $x^2 + y^2 = 4$, $z = x^2 + y^2 + 1$ and $z = 0$ and having density $\delta(x, y, z) = \sqrt{z}$.

12. (a) Find the first four nonzero terms of the Maclaurin series (i.e., the power series centered at zero) representation of the function $f(x) = e^{-x^2/2}$.

(b) Find the value $\int_0^1 f(x)\,dx$), accurate to the nearest hundredth. Justify that your answer has the required accuracy.