Name: * ANSWER KEY * 7/10/2018

Math 202 Calculus II Quiz 4

Please show all work and box your final answers. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. (30 points) Evaluate the integral.

$$\int_{\frac{1}{2}}^{1} \frac{1}{x^2\sqrt{x^2+4}}\,dx$$

LET $x = 2\tan\theta$ WHEN $x = 1$, $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

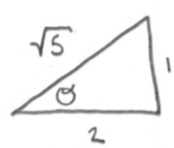
$dx = 2\sec^2\theta\, d\theta$ $x = \frac{1}{2}$, $\theta = \tan^{-1}\left(\frac{1}{4}\right)$

$$\rightsquigarrow \int_{\tan^{-1}\frac{1}{4}}^{\tan^{-1}\frac{1}{2}} \frac{2\sec^2\theta}{(2\tan\theta)^2(2\sec\theta)}\,d\theta = \frac{1}{4}\int_{\tan^{-1}\frac{1}{4}}^{\tan^{-1}\frac{1}{2}} \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \frac{1}{4}\int_{\tan^{-1}\frac{1}{4}}^{\tan^{-1}\frac{1}{2}} \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

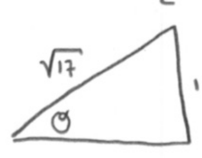
LET $u = \sin\theta$

$du = \cos\theta\, d\theta$

WHEN $\theta = \tan^{-1}\frac{1}{2} \longrightarrow$ 

$\longrightarrow \sin\theta = \frac{1}{\sqrt{5}}$

$\left(\tan\theta = \frac{1}{2}\right)$ $\left(\tan\theta = \frac{1}{4}\right)$

$\theta = \tan^{-1}\frac{1}{4} \longrightarrow$ 

$\longrightarrow \sin\theta = \frac{1}{\sqrt{17}}$

$$\rightsquigarrow \frac{1}{4}\int_{1/\sqrt{17}}^{1/\sqrt{5}} \frac{1}{u^2}\,du = -\frac{1}{4}\left[\frac{1}{u}\right]_{1/\sqrt{17}}^{1/\sqrt{5}} = \boxed{\frac{\sqrt{17}-\sqrt{5}}{4}}$$

2. (30 points) Evaluate the integral.

$$\int \frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)}\, dx$$

① PROPER RATIONAL FUNCTION ✓

② DENOMINATOR $Q(x)$ ALREADY FACTORED ✓

③ PARTIAL FRACTION DECOMPOSITION

$$\int \frac{x^2-2x-1}{(x-1)^2(x^2+1)}\,dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}\,dx$$

$$= A\ln|x-1| - \frac{B}{x-1} + \underbrace{\frac{C}{2}\ln|x^2+1|}_{\left(C\int \frac{x}{x^2+1}dx \quad \text{LET } u = x^2+1 \ldots\right)} + \underbrace{D\tan^{-1}x}_{\left(D\int \frac{1}{x^2+1}dx\right)} + E$$

NOTE THAT I CAN DO THE INTEGRATION WITHOUT KNOWING THE CONSTANTS A, B, C, D!

FINDING THE CONSTANTS:

$$x^2 - 2x - 1 = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2$$

LET $x = 1$ : $-2 = 2B \rightarrow \boxed{B = -1}$

THEN $x^2 - 2x - 1 = Ax^3 + Ax - Ax^2 - A - x^2 - 1 + Cx^3 - 2Cx^2 + Cx + Dx^2 - 2Dx + D$

$$0x^3 + 2x^2 - 2x + 0 = (A+C)x^3 + (-A-2C+D)x^2 + (A+C-2D)x + (-A+D)$$

① $A + C = 0$

② $-A - 2C + D = 2$

③ $A + C - 2D = -2$

④ $-A + D = 0$

① - ③ : $2D = D \rightarrow \boxed{D = 1}$

④ : $-A + 1 = 0 \rightarrow \boxed{A = 1}$

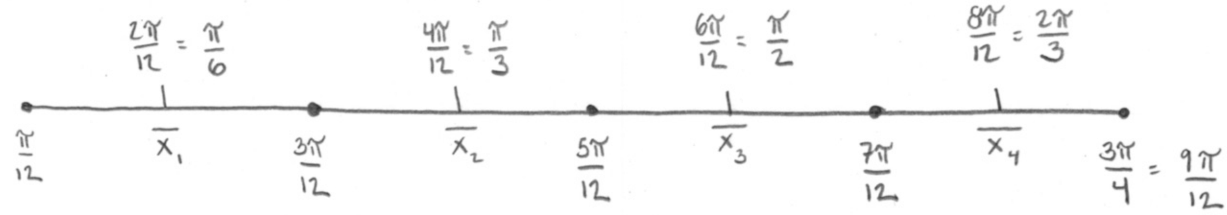
① : $1 + C = 0 \rightarrow \boxed{C = -1}$

∴ ANSWER: $\boxed{\ln|x-1| + \frac{1}{x-1} - \frac{1}{2}\ln|x^2+1| + \tan^{-1}x + C}$

3. (20 points) Use the Midpoint rule with $n = 4$ to approximate the integral.

$$\int_{\frac{\pi}{12}}^{\frac{3\pi}{4}} \sqrt{1+\cos x}\, dx$$

Evaluate the trigonometric functions and leave your answer in terms of square roots.




$$\Delta x = \frac{\frac{3\pi}{4} - \frac{\pi}{12}}{4} = \frac{1}{4}\left(\frac{9\pi - \pi}{12}\right) = \frac{\pi}{6}$$

$\frac{b-a}{n}$

$$M_4 = \Delta x \left( f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) \right)$$

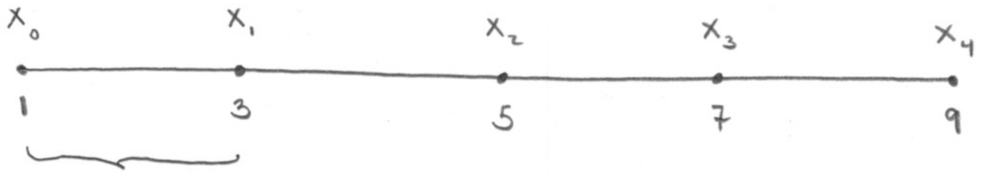
$$M_4 = \frac{\pi}{6}\left( \sqrt{1+\cos\frac{\pi}{6}} + \sqrt{1+\cos\frac{\pi}{3}} + \sqrt{1+\cos\frac{\pi}{2}} + \sqrt{1+\cos\frac{2\pi}{3}} \right)$$

$$M_4 = \boxed{\frac{\pi}{6}\left( \sqrt{1+\frac{\sqrt{3}}{2}} + \sqrt{1+\frac{1}{2}} + \sqrt{1} + \sqrt{1-\frac{1}{2}} \right)}$$

4. (20 points) Use Simpson's rule with $n = 4$ to approximate the integral.

$$\int_1^9 \ln(1+x^2)\,dx$$

Leave your answer in any form you like.




$$\Delta x = \frac{b-a}{n} = \frac{9-1}{4} = 2$$

$$S_4 = \frac{\Delta x}{3}\left( f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4) \right)$$

$$S_4 = \boxed{\frac{2}{3}\left( \ln(1+1^2) + 4\ln(1+3^2) + 2\ln(1+5^2) + 4\ln(1+7^2) + \ln(1+9^2) \right)}$$

or $\frac{2}{3}\left( \ln 2 + 4\ln 10 + 2\ln 26 + 4\ln 50 + \ln 82 \right)$