Name: * Answer Key * 5/8/2018

Math 202 Calculus II **Quiz 3**

All electronic devices must be turned off and put away (e.g. cellphones, calculators, etc.). Keep your eyes on your own paper and do not talk to other students. Answer all of the following questions. Show all of your work and put boxes around your final answers. Answers with no work shown will not receive credit. If you need more room, you may continue your work on the backs of the pages. Good luck!

1. (10 points) Use the *comparison test* to determine whether the following improper integral converges or diverges.

$$\int_1^\infty \frac{1}{x - e^{-x}}\, dx$$

FOR $x \geq 1$, $\quad 0 < x - e^{-x} < x$

THEREFORE, $\quad \dfrac{1}{x - e^{-x}} > \dfrac{1}{x}$.

SINCE $\displaystyle\int_1^\infty \frac{1}{x}\, dx$ DIVERGES BY p-TEST WITH $p = 1$,

THE COMPARISON TEST IMPLIES THAT $\displaystyle\int_1^\infty \frac{1}{x - e^{-x}}\, dx$ ALSO **DIVERGES.**

2. (10 points) Sketch the region enclosed by the curves

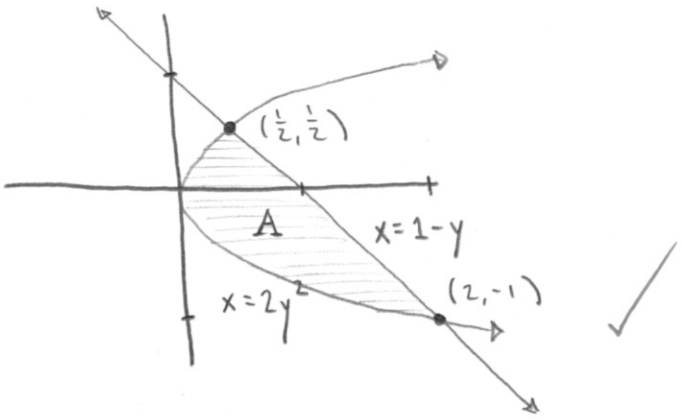
$$x = 2y^2 \quad \text{and} \quad x + y = 1,$$

and find find the area of the region.

INTERSECTION: $x + y = 1 \xrightarrow{x = 2y^2} 2y^2 + y = 1$

$2y^2 + y - 1 = 0 \longrightarrow (2y - 1)(y + 1) = 0$

$\left(\begin{matrix} y = -1 \\ x = 2 \end{matrix}\right), \left(\begin{matrix} y = \frac{1}{2} \\ x = \frac{1}{2} \end{matrix}\right)$



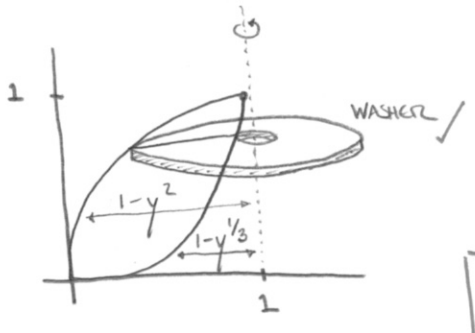
$$A = \int_{-1}^{1/2} (1 - y) - (2y^2)\, dy = y - \frac{1}{2}y^2 - \frac{2}{3}y^3 \Big|_{-1}^{1/2}$$

$$= \left(\frac{1}{2} - \frac{1}{8} - \frac{1}{12}\right) - \left(-1 - \frac{1}{2} + \frac{2}{3}\right)$$

$$= \frac{12 - 3 - 2 + 24 + 12 - 16}{24} = \frac{27}{24} = \boxed{\frac{9}{8}}$$

3. The region enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is rotated about the line $x = 1$ to produce a solid of revolution $\mathcal{S}$.

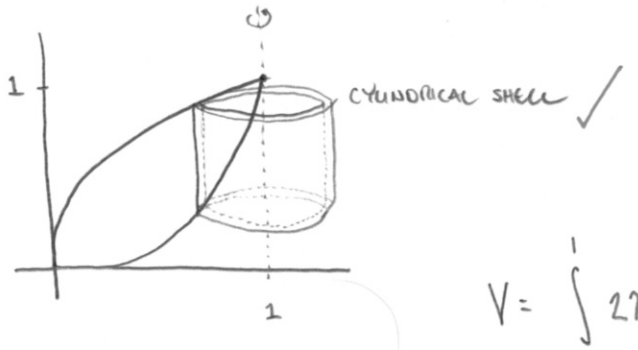
   (a) (5 points) Setup (*but do not evaluate*) an integral for the volume of $\mathcal{S}$ using the method of disks/washers. Include a sketch of a typical disk/washer.



$$V = \int_0^1 \pi \left[r_{out}^2 - r_{in}^2 \right] dy$$

$$V = \int_0^1 \pi \left[(1-y^2)^2 - (1-y^{1/3})^2 \right] dy$$

   (b) (5 points) Setup (*but do not evaluate*) an integral for the volume of $\mathcal{S}$ using the method of cylindrical shells. Include a sketch of a typical cylindrical shell.



$$V = \int_0^1 2\pi r h \, dx$$

$$V = \int_0^1 2\pi (1-x)(\sqrt{x} - x^3) \, dx$$

4. (10 points) Find the length of the following curve.

$$y = \frac{1}{12}x^3 + \frac{1}{x}, \quad 1 \le x \le 2$$

Let $f(x) = \frac{1}{12}x^3 + \frac{1}{x}$

$f'(x) = \frac{1}{4}x^2 - \frac{1}{x^2}$

$f'(x)^2 = \frac{1}{16}x^4 - \frac{1}{2} + \frac{1}{x^4}$

$$L = \int_1^2 \sqrt{1 + f'(x)^2}\, dx = \int_1^2 \sqrt{1 + \frac{1}{16}x^4 - \frac{1}{2} + \frac{1}{x^4}}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{16}x^4 + \frac{1}{2} + \frac{1}{x^4}}\, dx = \int \sqrt{\left(\frac{1}{4}x^2 + \frac{1}{x^2}\right)^2}\, dx$$

$$= \int_1^2 \frac{1}{4}x^2 + \frac{1}{x^2}\, dx = \frac{1}{12}x^3 - \frac{1}{x}\Bigg|_1^2$$

$$= \left(\frac{2}{3} - \frac{1}{2}\right) - \left(\frac{1}{12} - 1\right) = \frac{1}{6} + \frac{11}{12} = \boxed{\frac{13}{12}}$$

5. (10 points) A spring has a natural length of 20cm. If a 25-N force is required to keep it stretched to a length of 30cm (0.1m beyond its natural length), how much work is required to stretch it from 20cm to 25cm (0.05m beyond its natural length)?

$F = kx \longrightarrow 25 = k(.1) \Rightarrow k = 250$

$F = 250x$

$$W = \int_0^{.05} 250x\,dx = 125x^2 \Big|_0^{.05} = 125(.0025)\ \text{J}$$

$$= \frac{5}{16}\ \text{J} \text{ or } .3125\ \text{J}$$