All electronic devices must be turned off and put away (e.g. cellphones, calculators, etc.). Keep your eyes on your own paper and do not talk to other students. Answer all of the following questions. Show all of your work and put boxes around your final answers. If you need more room, you may continue your work on the backs of the pages. Good luck!

1. (a) (4 points) Let $f(x) = \dfrac{1 + 3x}{5 - 2x}$. Given that f is one-to-one, find $f^{-1}(x)$.

   (b) (4 points) Suppose g is the inverse function of f and $f(4) = 5$, $f'(4) = 2/3$. Find $g'(5)$.

2. Differentiate the function $y = \ln\sqrt{\dfrac{3x + 2}{3x - 2}}$ by first using log rules to simplify the function, and then differentiating.

3. Differentiate the function $y = x^{1/\ln x}$ using logarithmic differentiation.

4. (8 points) Scientists can determine the age of ancient objects by a method called *radiocarbon dating*. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, ^{14}C, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates ^{14}C through food chains. When a plant or animal dies, it stops replacing its carbon and the amount of ^{14}C begins to decrease through radioactive decay. Therefore, the level of radioactivity must also decay exponentially. A parchment fragment was discovered that had about 74% as much ^{14}C radioactivity as does plant material on Earth today. Estimate the age of the parchment.
i.e. *How long does it take a material with a half-life of 5730 years to decay to 74% of its original mass?*

5. (a) (4 points) Use the definitions of $\sinh x$ and $\cosh x$ to verify the identity $\cosh^2 x - \sinh^2 x = 1$.

(b) (8 points) Prove that $\dfrac{d}{dx}\left[\sinh^{-1} x\right] = \dfrac{1}{\sqrt{1+x^2}}$.

6. (8 points) Evaluate the limit $\lim\limits_{x \to \infty} \left(\dfrac{x}{x+1}\right)^x$.

7. (8 points) Evaluate the integral $\displaystyle\int \dfrac{10^{\sqrt{x}}}{\sqrt{x}}\, dx$.

8. Evaluate the following integrals.

(a) (8 points) $\displaystyle\int e^{2\theta} \sin(3\theta)\, d\theta$

(b) (8 points) $\displaystyle\int_0^{\pi/3} \tan^5 x \sec^4 x\, dx$

9. Evaluate the following integrals.

(a) (8 points) $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\, dt$

(b) (8 points) $\displaystyle\int \frac{1}{(x-1)^2(x+4)}\, dx$

10. Approximate the integral $\displaystyle\int_0^4 \sqrt{64 - x^3}\ dx$

    (a) (4 points) using the Midpoint Rule with $n = 4$ subintervals;

    (b) (4 points) using Simpson's Rule with $n = 4$ subintervals.

11. (8 points) Evaluate the following improper integral or show that it diverges.

$$\int_1^\infty \frac{\ln x}{x^2}\, dx$$