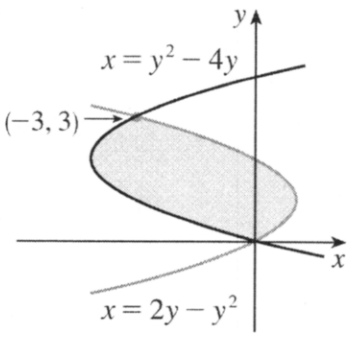
Name: * ANSWER KEY * 5/5/2016

Math 202 Quiz 3

Directions Answer all questions in the space provided. Show all work and box your final answers. Answers with no work shown will not receive full credit. Good luck!

1. (8 points) Find the area of the shaded region.



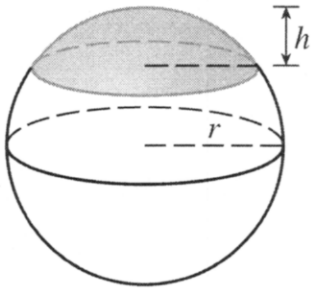
$$\text{AREA} = \int_0^3 \text{RIGHT} - \text{LEFT}\ dy$$

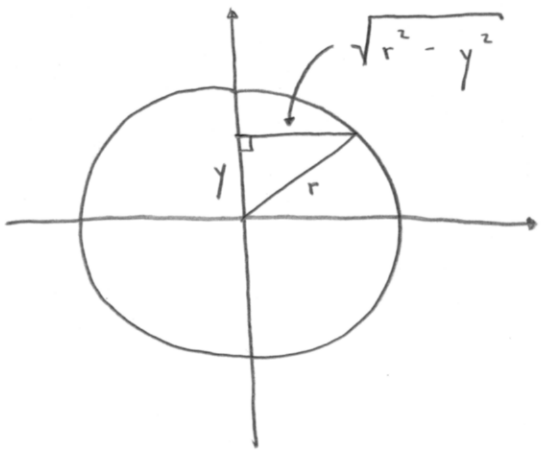
$$= \int_0^3 (2y - y^2) - (y^2 - 4y)\ dy$$

$$= \int_0^3 6y - 2y^2\ dy = 3y^2 - \frac{2}{3}y^3 \Big|_0^3$$

$$= 3(3)^2 - \frac{2}{3}(3)^3 - 0 = 27 - 18 = \boxed{9}$$

2. (8 points) Find the volume of a cap of a sphere with radius r and height h.





(METHOD OF DISKS)

$$\text{VOLUME} = \int_{r-h}^{r} \pi \left(\sqrt{r^2 - y^2}\right)^2 dy$$

$$= \pi \int_{r-h}^{r} r^2 - y^2 \, dy = \pi \left(r^2 y - \frac{1}{3} y^3 \right) \Bigg|_{r-h}^{r}$$

$$= \pi \left[\underbrace{\left(r^3 - \frac{1}{3} r^3 \right)} - \overbrace{\left(r^2 (r-h) - \frac{1}{3} (r-h)^3 \right)} \right]$$

$$= \pi \left[\frac{2}{3} r^3 - r^3 + r^2 h + \frac{1}{3} \left(r^3 - 3r^2 h + 3rh^2 - h^3 \right) \right]$$

$$= \pi \left[rh^2 - \frac{1}{3} h^3 \right] = \boxed{\pi h^2 \left(r - \frac{1}{3} h \right)}$$

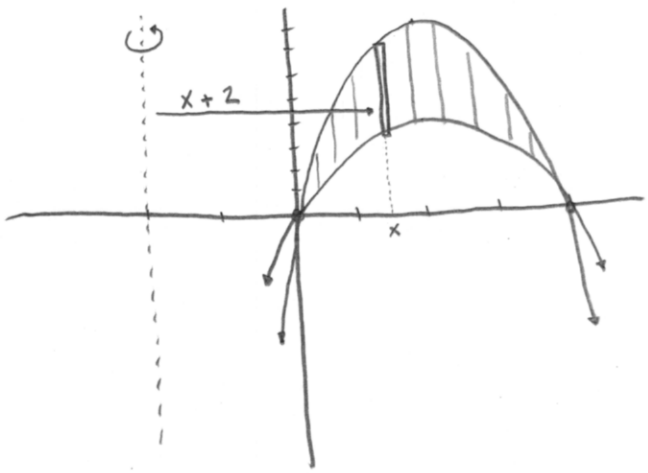
3. (8 points) Find the volume of the solid generated by rotating the region bounded by

$$y = 4x - x^2 \quad \text{and} \quad y = 8x - 2x^2$$

about the line $x = -2$.

$y = 4x - x^2 = x(4-x)$

$y = 8x - 2x^2 = 2x(4-x)$



(METHOD OF CYLINDRICAL SHELLS)

$$V = \int_0^4 2\pi(x+2)\left[(8x-2x^2)-(4x-x^2)\right]dx$$

$$= 2\pi\int_0^4 (x+2)(4x-x^2)\,dx = 2\pi\int_0^4 4x^2 - x^3 + 8x - 2x^2\,dx$$

$$= 2\pi\left[\frac{2}{3}x^3 - \frac{1}{4}x^4 + 4x^2\right]_0^4 = 2\pi\left(\frac{2}{3}(4)^3 - \frac{1}{4}(4)^4 + 4(4)^2\right)$$

$$= 32\pi\left(\frac{8}{3} - 4 + 4\right) = \boxed{\frac{256\pi}{3}}$$

4. (8 points) Find the length of the following curve.

$$y = \ln(\cos x), \quad 0 \le x \le \pi/3$$

$$L = \int_0^{\pi/3} \sqrt{1 + f'(x)^2}\, dx$$

$$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/3} \sec x\, dx = \ln(\sec x + \tan x)\Big|_0^{\pi/3}$$

$$= \ln(2 + \sqrt{3}) - \ln(1) = \boxed{\ln(2 + \sqrt{3})}$$

5. (8 points) A spring has a natural length of 2.5 meters. A force of 18 Newtons stretches the spring to a length of 5 meters. Determine how much work is done by stretching the spring 1.5 meters beyond its natural length.

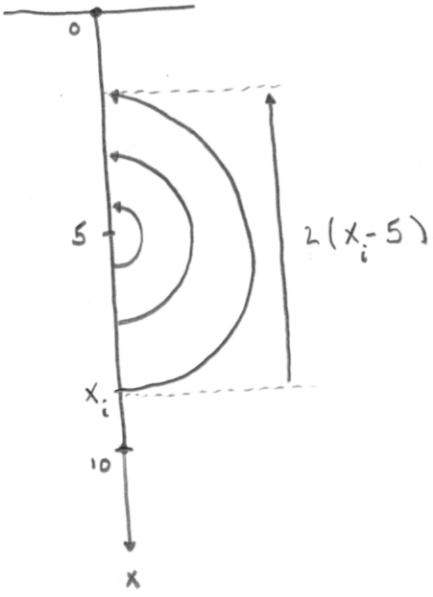
HOOKE'S LAW : $F = kx$

$$18 = 2.5k \longrightarrow k = \frac{18}{2.5} = \frac{36}{5} = 7.2$$

$$W = \int_0^{1.5} 7.2x\, dx = 3.6x^2\Big|_0^{1.5} = (3.6)(1.5)^2 - 0$$

$$= (3.6)(2.25) = \boxed{8.1 \text{ Joules}}$$

6. (8 points) A 10-ft chain weighs 25 lbs and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.



WEIGHT OF A SECTION OF CABLE Δx ft LONG

$$W \approx \sum_{i=1}^{n} \overbrace{\left(2.5 \text{ LBS/ft}\right)\Delta x} \cdot \underbrace{2(x_i - 5)}$$

DISTANCE THIS SECTION OF CABLE MUST TRAVEL

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} 5(x_i - 5)\Delta x$$

$$W = \int_5^{10} 5(x-5)\,dx = 5\left[\tfrac{1}{2}x^2 - 5x\right]_5^{10}$$

$$= 5\left[(50 - 50) - \left(\tfrac{25}{2} - 25\right)\right] = \boxed{\frac{125}{2} \text{ ft - LBS}}$$