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MATH 202

§ 9.4

$\boxed{1}$ $A = \int_a^b \frac{1}{2} r^2 \, d\theta = \int_0^{\pi/4} \frac{1}{2} (\sqrt{\theta})^2 \, d\theta = \frac{1}{4} \theta^2 \Big|_0^{\pi/4} = \boxed{\frac{\pi^2}{64}}$

$\boxed{3}$ $A = \int_{\pi/3}^{2\pi/3} \frac{1}{2} (\sin\theta)^2 \, d\theta = \frac{1}{4} \int_{\pi/3}^{2\pi/3} 1 - \cos 2\theta \, d\theta = \frac{1}{4} \Big[\theta - \frac{1}{2} \sin 2\theta \Big]_{\pi/3}^{2\pi/3}$

$= \frac{1}{4} \Big[\frac{\pi}{3} - \frac{1}{2} \Big(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \Big) \Big] = \boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{8}}$

$\boxed{5}$ $A = \int_0^{\pi} \frac{1}{2} \theta^2 \, d\theta = \frac{1}{6} \theta^3 \Big|_0^{\pi} = \boxed{\frac{\pi^3}{6}}$

$\boxed{7}$ $A = \int_{-\pi/2}^{\pi/2} \frac{1}{2} (4 + 3\sin\theta)^2 \, d\theta = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 16 + 24 \sin\theta + \underbrace{9 \sin^2\theta}_{\frac{9}{2}(1-\cos 2\theta)} \, d\theta$

$= \frac{1}{2} \int_{-\pi/2}^{\pi/2} \frac{41}{2} + 24 \sin\theta - \frac{9}{2} \cos 2\theta \, d\theta$

$= \frac{1}{2} \Big[\frac{41}{2} \theta - \underbrace{\cancel{24 \cos\theta}}_{0} - \underbrace{\cancel{\frac{9}{4} \sin 4\theta}}_{0} \Big]_{-\pi/2}^{\pi/2}$

$= \frac{1}{2} \Big[\frac{41}{2} \pi \Big] = \boxed{\frac{41}{4} \pi}$

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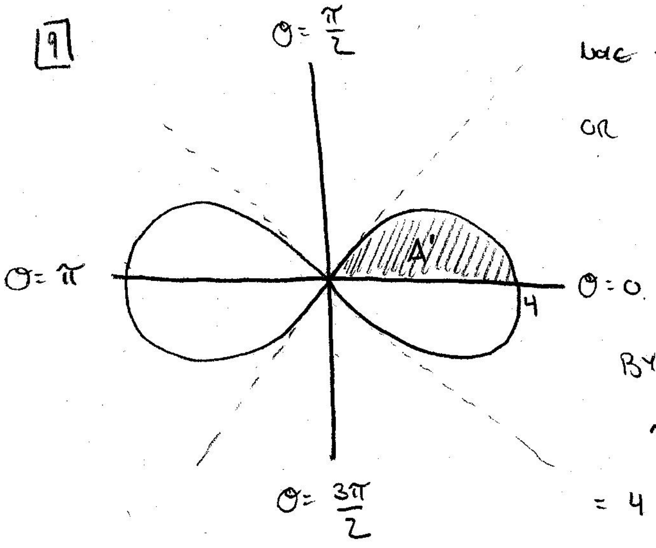
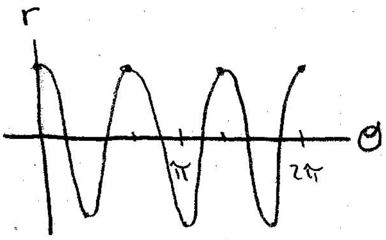


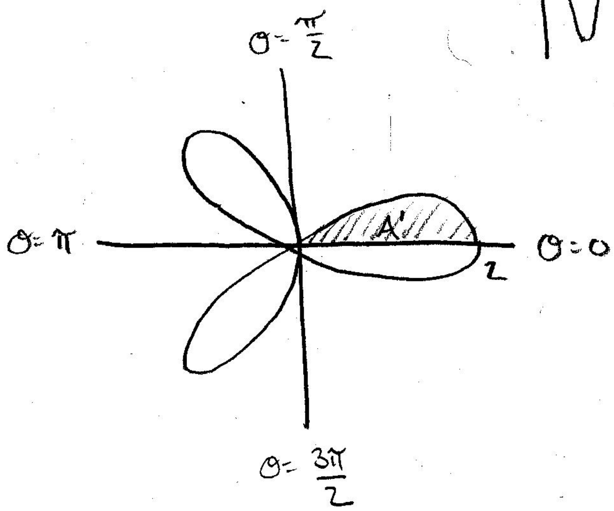
NOTE THAT THERE ARE NO POINTS WITH $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$ OR $\frac{5\pi}{4} < \theta < \frac{7\pi}{4}$ BECAUSE $r^2 \geq 0$.

BY SYMMETRY, $A = 4A' = 4\int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$

$= 4\int_0^{\pi/4} 2\cos 2\theta \, d\theta = 4\sin 2\theta \Big|_0^{\pi/4} = \boxed{4}$

11 $r = 2\cos(3\theta)$





BY SYMMETRY, $A = 6A' = 6\int_0^{\pi/6} 2\cos^2(3\theta)\, d\theta$

$= 6\int_0^{\pi/6} 1 + \cos(6\theta) = 6\left[\theta + \frac{1}{6}\sin(6\theta)\right]_0^{\pi/6}$

$= \boxed{\pi}$

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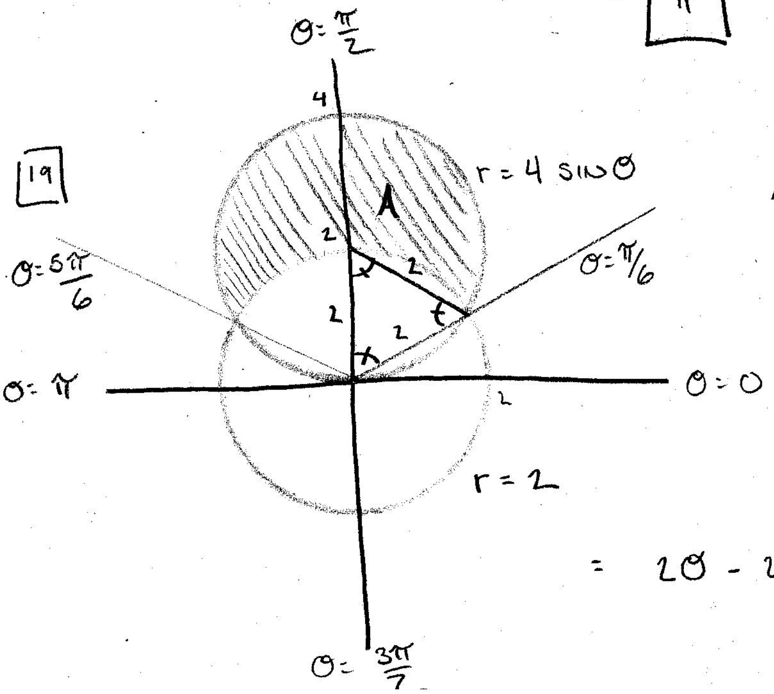

$A = \int_{\pi/6}^{5\pi/6} \frac{1}{2}(4\sin\theta)^2 - \frac{1}{2}(2)^2 \, d\theta$

$= \int_{\pi/6}^{5\pi/6} 4(1 - \cos 2\theta) - 2 \, d\theta$

$= 2\theta - 2\sin 2\theta \Big|_{\pi/6}^{5\pi/6} = \boxed{\frac{4\pi}{3} + 2\sqrt{3}}$

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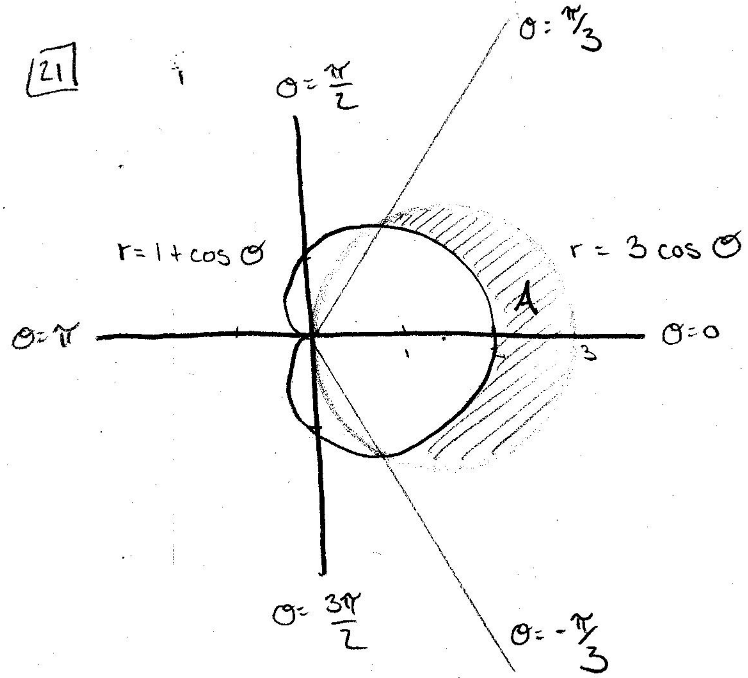

INTERSECTION :

$$3\cos\theta = 1 + \cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \pm\frac{\pi}{3}$$

$$A = \int_{-\pi/3}^{\pi/3} \frac{1}{2}(3\cos\theta)^2 - \frac{1}{2}(1+\cos\theta)^2 \, d\theta = \frac{1}{2}\int_0^{\pi/3} 9\cos^2\theta - 1 - 2\cos\theta - \cos^2\theta \, d\theta$$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3} \underbrace{8\cos^2\theta}_{4(1+\cos 2\theta)} - 1 - 2\cos\theta \, d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3} 3 + 4\cos 2\theta - 2\cos\theta \, d\theta$$

$$= \frac{1}{2}\Big[3\theta + 2\sin 2\theta - 2\sin\theta\Big]_{-\pi/3}^{\pi/3} = \frac{1}{2}\Big(2\pi + 2(\sqrt{3}) - 2(\sqrt{3})\Big) = \boxed{\pi}$$

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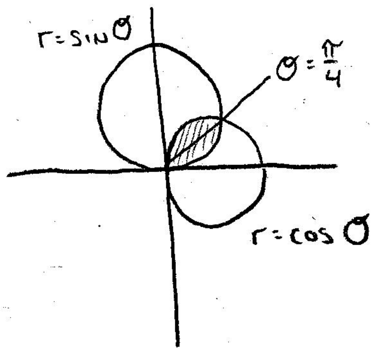

$$A = 2\int_0^{\pi/4} \frac{1}{2}\sin^2\theta \, d\theta = \int_0^{\pi/4} \sin^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} 1 - \cos 2\theta \, d\theta = \frac{1}{2}\Big[\theta - \frac{1}{2}\sin 2\theta\Big]_0^{\pi/4}$$

$$= \boxed{\frac{\pi}{8} - \frac{1}{4}}$$

29 $r = \cos\theta$, $r = 1 - \cos\theta$

$\to \cos\theta = 1 - \cos\theta \longrightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \pm\frac{\pi}{3} + 2n\pi$

FOR $\theta = \pm\frac{\pi}{3}$, $r = \cos\pm\frac{\pi}{3} = \frac{1}{2}$ $\Rightarrow$ INTERSECTION POINTS

$\boxed{\left(\frac{1}{2}, \frac{\pi}{3}\right) \ \& \ \left(\frac{1}{2}, -\frac{\pi}{3}\right)}$

31 $r = \sin\theta$, $r = \sin 2\theta$

$\to \sin\theta = \sin 2\theta = 2\sin\theta\cos\theta$

$2\sin\theta\cos\theta - \sin\theta = \sin\theta(2\cos\theta - 1) = 0$

$\Rightarrow$ EITHER $\sin\theta = 0$ OR $\cos\theta = \frac{1}{2}$

$\theta = 0, \pi$ $\quad\quad$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$r = 0$ $\quad$ $r = 0$ $\quad\quad$ $r = \frac{\sqrt{3}}{2}$ $\quad$ $r = -\frac{\sqrt{3}}{2}$

(SAME POINT! ORIGIN)

$$\therefore \boxed{\underset{(1)}{(0,0)} \ \& \ \underset{(2)}{\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right)} \ \& \ \left(-\frac{\sqrt{3}}{2}, \frac{5\pi}{3}\right) = \underset{(3)}{\left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3}\right)}}$$

33 $$L = \int_0^{\pi/3} \sqrt{(3\sin\theta)^2 + (3\cos\theta)^2}\, d\theta = 3\int_0^{\pi/3} d\theta = \boxed{\pi}$$

35 $$L = \int_0^{2\pi} \sqrt{(\theta^2)^2 + (2\theta)^2}\, d\theta = \int_0^{2\pi} \theta\sqrt{\theta^2 + 4}\, d\theta$$

LET $u = \theta^2 + 4$
$du = 2\theta\, d\theta$

$$= \frac{1}{2}\int_4^{4\pi^2+4} \sqrt{u}\, du = \frac{1}{3} u^{3/2}\Big|_4^{4\pi^2+4} = \frac{1}{3}\left[(4\pi^2+4)^{3/2} - 4^{3/2}\right] = \frac{4^{3/2}}{3}\left[(\pi^2+4)^{3/2} - 1\right]$$

$$= \boxed{\frac{8}{3}\left[(\pi^2+4)^{3/2} - 1\right]}$$