JOHN ADAMSKI

7/16/2012

MATH 202

§ 9.2

1. $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \boxed{\frac{-5}{1-3t^2}}$$

3. $$\left.\frac{dy}{dx}\right|_{t=-1} = \left.\frac{3t^2+1}{4t^3}\right|_{t=-1} = \frac{4}{-4} = \boxed{-1}$$

OH! THIS IS NOT THE ANSWER!

THIS IS JUST THE SLOPE OF THE TANGENT LINE.

WHEN $t=-1$, $x=(-1)^4+1=2$ & $y=(-1)^3+(-1)=-2$

$\therefore\ y+2=-(x-2)$ OR $\boxed{y=-x}$

5. WHEN $t=1$, $x=e^{\sqrt{1}}=e$, $y=1-\ln(1^2)=1$

AND $$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{1-\frac{2}{t}}{e^{\sqrt{t}}\left(\frac{1}{2\sqrt{t}}\right)}\right|_{t=1} = \frac{-1}{\frac{e}{2}} = -\frac{2}{e}$$

$\therefore\ y-1=-\frac{2}{e}(x-e)$ OR $\boxed{y=-\frac{2}{e}x+3}$

7 (a) $x = e^t = 1 \Rightarrow t = 0$

WHEN $t = 0$, $\frac{dy}{dx} = \left.\frac{2(t-1)}{e^t}\right|_{t=0} = \frac{-2}{1} = -2$

$\therefore\ y - 1 = -2(x-1)$ OR $\boxed{y = -2x + 3}$

(b) $x = e^t$; $y = (t-1)^2$

$\downarrow$

$t = \ln x$ $\quad\therefore\ y = (\ln x - 1)^2$

$y' = 2(\ln x - 1)\frac{1}{x}$

AT $(1,1)$ WE HAVE $y'(1) = 2(\ln(1) - 1)\frac{1}{1} = -2$

$\therefore\ y - 1 = -2(x-1) \Rightarrow \boxed{y = -2x + 3}$ ✓

9 $x = 4 + t^2$, $y = t^2 + t^3$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t + 3t^2}{2t} = \frac{2t}{2t} + \frac{3t^2}{2t} = \boxed{1 + \frac{3}{2}t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{3/2}{2t} = \boxed{\frac{3}{t}}$$

CONCAVE UP MEANS $\frac{d^2y}{dx^2} > 0$, SO $\boxed{t > 0}$

11 $x = t - e^t$, $y = t + e^{-t}$

$$\frac{dy}{dx} = \frac{1 - e^{-t}}{1 - e^t} = \frac{1 - \frac{1}{e^t}}{1 - e^t} = \frac{\frac{e^t - 1}{e^t}}{1 - e^t} = \boxed{-e^{-t}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(-e^{-t}\right)}{\frac{dx}{dt}} = \boxed{\frac{e^{-t}}{1 - e^t}}$$

ALWAYS POS.

POS FOR $\boxed{t < 0}$

13 $x = 10 - t^2$, $y = t^3 - 12t$

$x'(t) = -2t$, $y'(t) = 3t^2 - 12$

NOTE: $y'(0) = -12 \neq 0$ ✓

VERTICAL TANGENT WHEN $x'(t) = 0$, $y'(t) \neq 0$

i.e. $-2t = 0 \Rightarrow t = 0$

i.e. @ $\boxed{(10, 0) \text{ VERT. TANGENT}}$

HORIZONTALE TANGENT WHEN $y'(t) = 0$, $x'(t) \neq 0$

i.e. $3t^2 - 12 = 0$

$t^2 - 4 = 0$

$(t+2)(t-2) = 0$

$t = \pm 2$

WHEN $t = -2$, $x'(-2) = 4 \neq 0$

$t = 2$, $x'(2) = -4 \neq 0$

$\therefore$ $(x(-2), y(-2))$ & $(x(2), y(2))$

$\boxed{(6, 16), (6, -16) \text{ HOR. TANGENT}}$

15 $x = 2\cos\theta \qquad y = \sin 2\theta$

HORIZONTILE TANGENT WHEN $y'(\theta) = 0$ & $x'(\theta) \neq 0$

$y'(\theta) = 2\cos(2\theta) = 0 \Rightarrow \cos(2\theta) = 0$

$\Rightarrow 2\theta = \frac{\pi}{2} + n\pi \Rightarrow \theta = \frac{\pi}{4} + \frac{n}{2}\pi$

i.e. $\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \ldots$

$x'(\theta) = -2\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi$

i.e. $\theta = 0, \pm\pi, \pm 2\pi, \ldots$

NOTE: NEVER AT THE SAME TIME!

$\therefore$ HOR. TANGENTS @ $y = \sin\left(2\left(\frac{\pi}{4} + \frac{n}{2}\pi\right)\right) = \pm 1$

$x = 2\cos\left(\frac{\pi}{4} + \frac{n}{2}\pi\right) = \pm\frac{\sqrt{2}}{2}$

$\Rightarrow$ FOUR POINTS $\boxed{\left(\pm\frac{\sqrt{2}}{2}, \pm 1\right) \text{ HORIZONTILE TANGENTS}}$

VER. TANGENTS @ $x = 2\cos(n\pi) = \pm 2$

$y = \sin(2n\pi) = 0$

$\Rightarrow$ TWO POINTS $\boxed{(\pm 2, 0) \text{ VERTICAL TANGENTS}}$

33

$$L = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$L = \int_1^2 \sqrt{(1-2t)^2 + (2\sqrt{t})^2}\, dt$$

$$= \int_1^2 \sqrt{1 - 4t + 4t^2 + 4t}\, dt$$

$$\boxed{L = \int_1^2 \sqrt{1+4t^2}\, dt}$$

35

$$L = \int_0^{2\pi} \sqrt{(1-\sin t)^2 + (1-\cos t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\sin t + \sin^2 t + 1 - 2\cos t + \cos^2 t}\, dt$$

$$\boxed{L = \int_0^{2\pi} \sqrt{3 - 2(\sin t + \cos t)}\, dt}$$

$\boxed{37}$ $$L = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{36t^2 + 36t^4}\, dt = \int_0^1 \sqrt{(36t^2)(1+t^2)}\, dt$$

$$= 6\int_0^1 t\sqrt{1+t^2}\, dt = 3\int_1^2 \sqrt{u}\, du = 3 \cdot \frac{2}{3} u^{3/2} \Big|_1^2$$

$$= 2(2\sqrt{2} - 1) = \boxed{4\sqrt{2} - 2}$$

$\boxed{39}$ $x = \dfrac{1}{1+t}$ $\qquad y = \ln(1+t)$

$$x'(t) = -(1+t)^{-2} = \frac{-1}{(1+t)^2} \; ; \; y'(t) = \frac{1}{1+t}$$

$$L = \int_0^2 \sqrt{\left(\frac{-1}{(1+t)^2}\right)^2 + \left(\frac{1}{1+t}\right)^2}\, dt$$

HOLD ON TO YOUR HAT!!

$$= \int_0^2 \sqrt{\frac{1}{(1+t)^4} + \frac{1}{(1+t)^2}}\, dt = \int_0^2 \sqrt{\frac{1 + (1+t)^2}{(1+t)^4}}\, dt$$

$$= \int_0^2 \frac{\sqrt{1 + (1+t)^2}}{(1+t)^2}$$

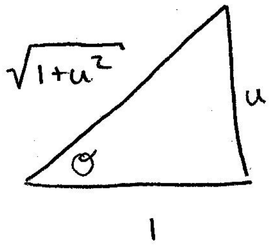
LET $u = (1+t)$

$du = dt$

$$\longrightarrow \int_1^3 \frac{\sqrt{1+u^2}}{u^2}$$

LET $u = \tan\theta$

$du = \sec^2\theta \, d\theta$



$$\hookrightarrow \int \frac{\sec\theta}{\tan^2\theta} \sec^2\theta \, d\theta = \int \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos^3\theta} \, d\theta$$

$$= \int \frac{\cos\theta}{\sin^2\theta \cos^2\theta} \, d\theta = \int \frac{\cos\theta}{\sin^2\theta(1-\sin^2\theta)} \, d\theta$$

LET $w = \sin\theta$

$dw = \cos\theta \, d\theta$

$$\longrightarrow \int \frac{1}{w^2(1-w^2)} \, dw$$

$$\frac{1}{w^2(1-w)(1+w)} = \frac{A}{w} + \frac{B}{w^2} + \frac{C}{1-w} + \frac{D}{1+w}$$

$$1 = Aw(1-w)(1+w) + B(1-w)(1+w) + Cw^2(1+w) + Dw^2(1-w)$$

$w=0 \Rightarrow 1 = B$ THEN $1 = (-A + C - D)w^3 + \ldots\ldots$

$w=-1 \Rightarrow 1 = 2D$ $1 = -Aw^3 + \ldots \Rightarrow A = 0$

$w = 1 \Rightarrow 1 = 2C$ (EQUATING COEFFICIENTS)

$$= \int \frac{1}{w^2} + \frac{1}{2(1-w)} + \frac{1}{2(1+w)} \, dw$$

$$= \frac{-1}{w} - \frac{1}{2}\ln|1-w| + \frac{1}{2}\ln|1+w|$$

$$\rightarrow \frac{-1}{\sin\theta} - \frac{1}{2}\ln|1-\sin\theta| + \frac{1}{2}\ln|1+\sin\theta|$$

$$\rightarrow \left[\frac{-\sqrt{1+u^2}}{u} - \frac{1}{2}\ln\left|1-\frac{u}{\sqrt{1+u^2}}\right| + \frac{1}{2}\ln\left|1+\frac{u}{\sqrt{1+u^2}}\right| \right]_1^3$$

log LAWS

$$= \left[\frac{-\sqrt{1+u^2}}{u} + \frac{1}{2}\ln\left| \frac{1+\frac{u}{\sqrt{1+u^2}}}{1-\frac{u}{\sqrt{1+u^2}}} \right| \right]_1^3$$

$$= \left[\frac{-\sqrt{1+u^2}}{u} + \frac{1}{2}\ln\left| \frac{\sqrt{1+u^2}+u}{\sqrt{1+u^2}-u} \right| \right]_1^3$$

RATIONALIZE DENOM.

$$= \left[\frac{-\sqrt{1+u^2}}{u} + \frac{1}{2}\ln\left| \frac{(\sqrt{1+u^2}+u)^2}{(1+u^2)-u^2} \right| \right]_1^3$$

$$= \left[\frac{-\sqrt{1+u^2}}{u} + \ln\left|\sqrt{1+u^2} + u\right| \right]_1^3$$

$$= \left[\frac{-\sqrt{10}}{3} + \ln(\sqrt{10}+3) \right] - \left[-\sqrt{2} + \ln(\sqrt{2}+1) \right]$$

$$= \boxed{\sqrt{2} - \frac{\sqrt{10}}{3} + \ln \frac{\sqrt{10}+3}{\sqrt{2}+1}}$$

... OR USE #24 IN TABLE OF INTEGRALS

IN THE BACK OF THE BOOK :)