MATH 202

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§7.5

1 $\displaystyle\int_0^9 \frac{10}{(1+x)^2}\,dx$ Let $u = 1+x$, $du = dx$ $\left\{ \begin{array}{l} x = 0 \Rightarrow u = 1 \\ x = 9 \Rightarrow u = 10 \end{array} \right.$

$$\rightarrow \int_1^{10} \frac{10}{u^2}\,dx = -10u^{-1}\Big|_1^{10} = -1 + 10 = \boxed{9 \text{ FT-LBS.}}$$

3 $\displaystyle W = \int_0^8 F(x)\,dx = \text{AREA UNDER GRAPH} = (\text{AREA OF } \triangle) + (\text{AREA OF } \square)$

$$= \frac{1}{2}(4)(30) + (4)(30) = 60 + 120 = \boxed{180\text{J}}$$

4 $\displaystyle W = \int_0^{18} F(x)\,dx \approx \frac{\Delta x}{3}\Big[f(0) + 4f(3) + 2f(6) + 4f(9) + 2f(12) + 4f(15) + f(18)\Big]$

$$= \frac{3}{3}\Big[9.8 + 4(9.1) + 2(8.5) + 4(8.0) + 2(7.7) + 4(7.5) + 7.4\Big]$$

$$= \boxed{148 \text{ J}}$$

$\boxed{5}$ LET x = DISTANCE BEYOND NATURAL LENGTH THE SPRING IS STRETCHED

HOOKE'S LAW : $F = kx$; $4 \text{ in} = \frac{1}{3}$ FT.

$6 \text{ in} = \frac{1}{2}$ FT

$10 = k\left(\frac{1}{3}\right) \Rightarrow \underline{\underline{k = 30}}$

$$W = \int_0^{1/2} 30x\,dx = 15x^2\Big|_0^{1/2} = \frac{15}{4} \text{ J}$$

$$\boxed{W = 15/4 \text{ J}}$$

$F = kx$ (HOOKE'S LAW)

DISTANCE IS MEASURED IN METERS !

$\boxed{7}$ $W = Fd \longrightarrow 2 = \int_0^{.12} kx\,dx = \frac{k}{2}x^2\Big|_0^{.12} = .0072k$

$$\Rightarrow k = \frac{2}{.0072} \approx 278$$

(a) $W = \int_{.05}^{.1} \frac{1}{.0036}x\,dx = \frac{1}{.0072}x^2\Big|_{.05}^{.1} = \frac{.01 - .0025}{.0072}$

$$= \frac{.0075}{.0072} = \boxed{\frac{25}{24} \text{ J} \approx 1.04 \text{ J}}$$

(b) $F = kx \longrightarrow 30 = \frac{1}{.0036}x \Rightarrow x = \boxed{.108 \text{ m} = 10.8 \text{ cm}}$

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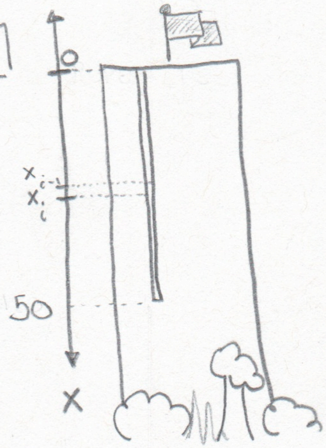
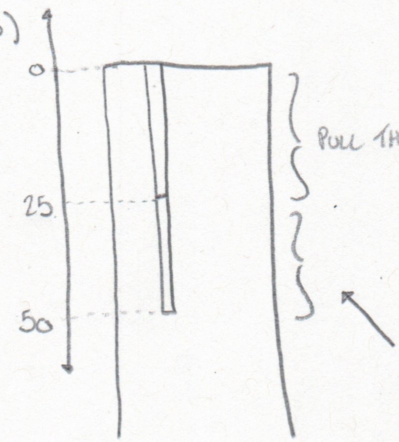


$$W_i = Fd = \left(\tfrac{1}{2}\ \text{LB/FT}\right)(\Delta x\ \text{FT})(x_i)$$

$$W \approx \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} \tfrac{1}{2} x_i \, \Delta x$$

$$W = \int_0^{50} \tfrac{1}{2} x \, dx = \tfrac{1}{4} x^2 \Big|_0^{50} = \frac{2500}{4} = \boxed{625 \text{ FT-LBS.}}$$

(b)



PULL THIS PART IN AS IN PART (a) $\quad W = \int_0^{25} \tfrac{1}{2} x \, dx = \frac{x^2}{4} \Big|_0^{25}$

$$= \frac{625}{4} = 156.25 \text{ J}$$

THIS PART WEIGHS $\tfrac{1}{2}(25) = 12.5$ LBS.

& IS PULLED THROUGH 25 FT.

HENCE $W = (12.5)(25) = 312.5$ J

TOTAL WORK $= 156.25 + 312.5 = \boxed{468.75 \text{ J}}$

WHICH IS $\tfrac{3}{4}$ THE WORK REQUIRED TO PULL IN THE <u>ENTIRE</u> ROPE!

11 WORK TO PULL IN CABLE $= \int_0^{500} 2x \, dx = x^2 \Big|_0^{500} = 250{,}000$ FT-LBS.

WORK TO PULL UP COAL $= (800 \text{ LBS})(500 \text{ FT.}) = 400{,}000$ FT. LBS.

TOTAL WORK $= 250{,}000 + 400{,}000 = \boxed{650{,}000 \text{ FT. LBS.}}$

12 SUPPOSE WE START LIFTING THE BUCKET WHEN TIME $t = 0$.

WE BREAK THE WORK INTO TINY SUBINTERVALS, SHORT ENOUGH SO THE WEIGHT OF THE BUCKET + WATER IS APPROXIMATELY CONSTANT DURING EACH (TIME) SUBINTERVAL.

$W_i = F_i d$ WHERE $F_i = (4 \text{ LBS BUCKET}) + (40 - .2t \text{ LBS WATER})$

$= 44 - .2t \text{ (LBS)}$

AND $d = (2 \text{ FT/s})\, \Delta t = 2\Delta t \text{ FT.}$

$$\therefore W \approx \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} (44 - .2t) \cdot 2\Delta t = (88 - .4t)\Delta t$$

$$W = \int_0^{40} (88 - .4t)\, dt$$

BECAUSE IT TAKES 40 SEC TO LIFT THE BUCKET 80 FT @ 2FT/SEC.

$$= 88t - .2t^2 \Big|_0^{40} = 3520 - 320 = \boxed{3200 \text{ FT-LBS.}}$$

ALTERNATIVELY, THE BUCKET + WATER WEIGH 44 LBS. AT THE BOTTOM, AND WEIGH $44 - 40(.2) = 44 - 8 = 36$ LBS. AT THE TOP.

$\therefore$ THE AVERAGE WEIGHT IS 40 LBS.

$$W = Fd = (40 \text{ LBS})(80 \text{ FT.}) = \boxed{3200 \text{ FT-LBS.}}$$

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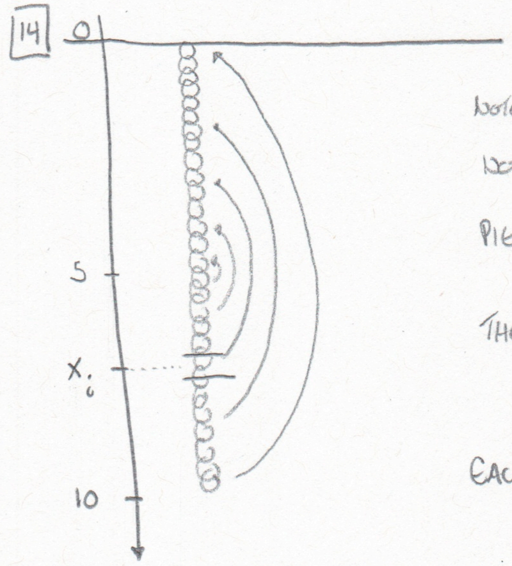

NOTE THAT THE TOP HALF OF THE CHAIN DOES NOT MOVE. BREAK THE BOTTOM HALF INTO PIECES.

THE CHAIN WEIGHS $\frac{25 \text{ LBS}}{10 \text{ FT}} = 2.5$ LBS/FT.

EACH PIECE IS Δx LONG & WEIGHS $2.5\,\Delta x$ LBS.

THE SUBINTERVAL x_i BELOW THE CIELING GETS LIFTED $2(x_i - 5)$ FT. HENCE $W_i = (2.5\,\Delta x \text{ LBS}) \cdot 2(x_i - 5 \text{ FT})$

$$= 5(x_i - 5)\,\Delta x \text{ N}$$

$$\therefore\ W \approx \sum_{i=1}^{n} 5(x_i - 5)\,\Delta x$$

$$W = \int_5^{10} 5(x-5)\,dx = 5\left(\tfrac{1}{2}x^2 - 5x\right)\Big|_5^{10} = 5\left[(50-50) - \left(\tfrac{25}{2} - 25\right)\right]$$

$$= \boxed{\frac{125}{2} \text{ FT.-LBS.}}$$

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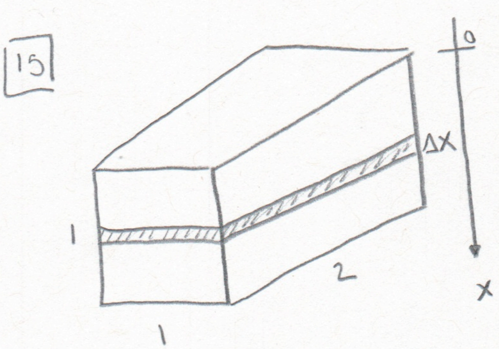

A LAYER OF WATER Δx THICK HAS MASS

$(1000 \text{ kg/m}^2) \cdot (1\text{m})(2\text{m})(\Delta x \text{ m})$

$= 2000\,\Delta x$ kg $\Rightarrow$ WEIGHT $9.8 \cdot 2000\,\Delta x$ N

AND MUST BE LIFTED x_i m TO THE TOP OF THE TANK. THUS $W \approx \sum_{i=1}^{n} 19600\, x_i\,\Delta x$

ONLY EMPTY HALF THE TANK!

$$\text{AND } W = \int_0^{1/2} 19600x\,dx = 9800x^2 \Big|_0^{1/2} = \frac{9800}{4} = \boxed{2450 \text{ J}}$$

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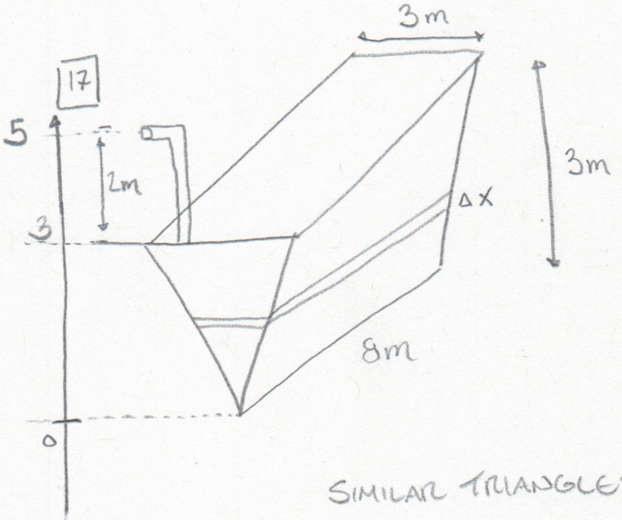

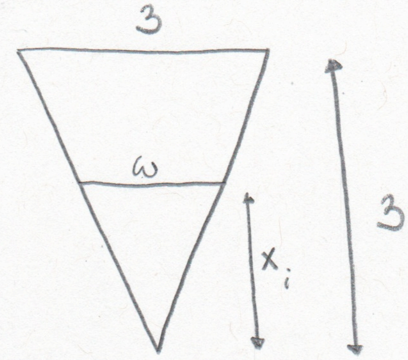

$$\text{SIMILAR TRIANGLES} \rightarrow \frac{w}{x_i} = \frac{3}{3} \rightarrow W = x_i$$

VOLUME OF A LAYER OF WATER $= (x_i)(8)(\Delta x)\ m^3 = 8x_i \Delta x\ m^3$

MASS OF A LAYER OF WATER $= (1000\ kg/m^3)(8x_i\Delta x\ m^3)$

$= 8000 x_i \Delta x\ N$

WEIGHT OF A LAYER OF WATER $= (9.8\ m/s^2)(8000\, x_i\, \Delta x\ N)$

$= 78400\, x_i\, \Delta x\ N$

DISTANCE THE LAYER MUST TRAVEL $= 2 + (3 - x_i)\ m = 5 - x_i\ m$

$$\therefore W \approx \sum_{i=1}^{n} (5 - x_i) \cdot 78400\, x_i\, \Delta x$$

$$W = \int_0^3 78400(5x - x^2)\,dx = 78400\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^3$$

$$= 78400\left(\frac{45}{2} - 9\right) = \boxed{1{,}058{,}400 \text{ J}}$$

(b) $$470000 = \int_0^d 78400(5x - x^2)\,dx = 78400\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^d$$

$$\frac{470000}{78400} = \frac{5}{2}d^2 - \frac{1}{3}d^3$$

YOU CAN'T SOLVE A CUBIC EQ. WITHOUT A CALCULATOR $\because$

($d \approx 2\,m$ I GUESS.)