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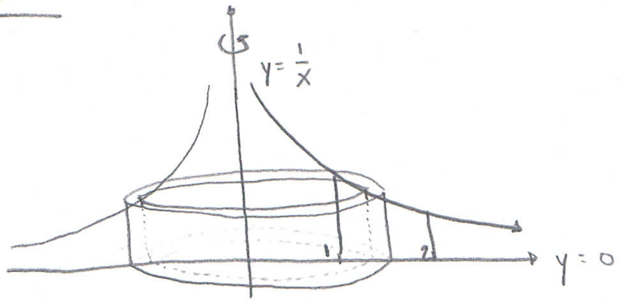
MATH 202

7/3/2012

§ 7.3

[3]



$$V = \int_1^2 2\pi x\left(\frac{1}{x}\right)dx = 2\pi\int_1^2 dx$$

$$= 2\pi x\Big|_1^2 = 2\pi(2-1) = \boxed{2\pi}$$

[7] $y = 4(x-2)^2$

$y = x^2 - 4x + 7 = (x-2)^2 + 3$

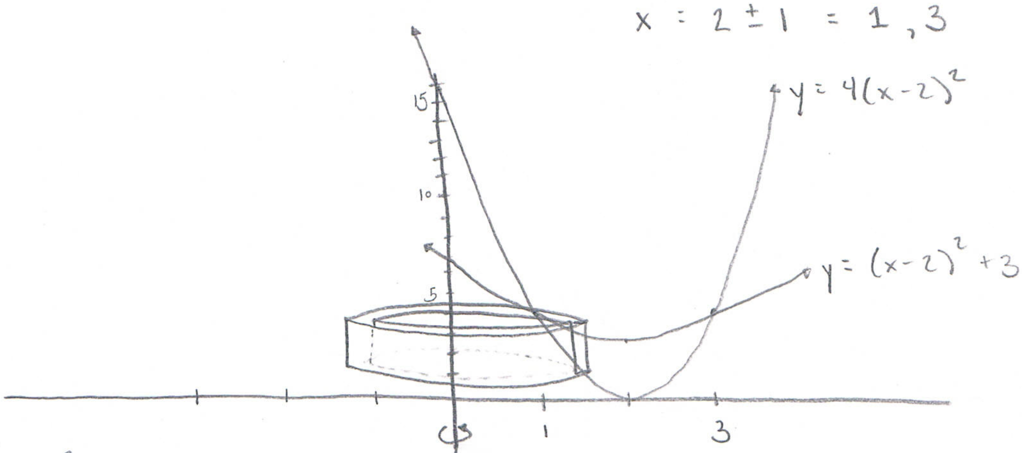
THESE CURVES INTERSECT WHERE $4(x-2)^2 = (x-2)^2 + 3$

$$3(x-2)^2 = 3$$

$$(x-2)^2 = 1$$

$$x - 2 = \pm 1$$

$$x = 2 \pm 1 = 1, 3$$



$$V = \int_1^3 2\pi x\left[\left((x-2)^2+3\right) - \left(4(x-2)^2\right)\right]dx$$

$$= \int_1^3 2\pi x\left[-3(x-2)^2 + 3\right]dx = 6\pi\int_1^3 -x^3 + 4x^2 - 3x\,dx$$

$$= 6\pi\left[-\frac{1}{4}x^4 + \frac{4}{3}x^3 - \frac{3}{2}x^2\right]_1^3 = 6\pi\left[\left(-\frac{81}{4} + \frac{108}{3} - \frac{27}{2}\right) - \left(-\frac{1}{4} + \frac{4}{3} - \frac{3}{2}\right)\right]$$

$$= 6\pi\left[-20 + 34\tfrac{2}{3} - 12\right] = 6\pi\left(\frac{8}{3}\right) = \boxed{16\pi}$$

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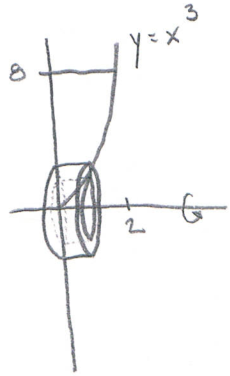

$$y = x^3 \iff x = y^{1/3}$$

$$V = \int_0^8 2\pi y \left(y^{1/3}\right) dy = 2\pi \int_0^8 y^{4/3}\, dy$$

$$= 2\pi \left(\tfrac{3}{7}\right) y^{7/3} \Big|_0^8 = \frac{6\pi}{7} \cdot 2^7 = \frac{6 \cdot 128\pi}{7} = \boxed{\frac{768\pi}{7}}$$

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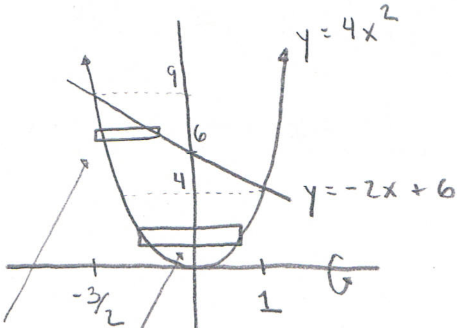


NOTE: THESE SHELLS COME IN TWO TYPES!

THESE CURVES INTERSECT WHEN

$$4x^2 = -2x + 6$$
$$4x^2 + 2x - 6 = 0$$
$$2x^2 + x - 3 = 0$$
$$(2x+3)(x-1) = 0$$
$$x = -\frac{3}{2} \;;\; x = 1$$

$$y = 4x^2 \iff x = \pm \frac{1}{2}\sqrt{y} \quad ; \quad y = -2x + 6 \iff x = -\frac{1}{2}y + 3$$

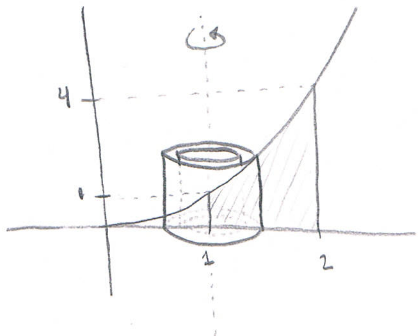
$$V = \int_4^9 2\pi y \left[\left(-\tfrac{1}{2}y + 3\right) - \left(-\tfrac{1}{2}\sqrt{y}\right)\right] dy + \int_0^4 2\pi y \left[2\left(\tfrac{1}{2}\sqrt{y}\right)\right] dy$$

$$= 2\pi \int_4^9 -\tfrac{1}{2}y^2 + 3y + \tfrac{1}{2}y^{3/2}\, dy + 2\pi \int_0^4 y^{3/2}\, dy$$

$$= 2\pi \left[\left[-\tfrac{1}{6}y^3 + \tfrac{3}{2}y^2 + \tfrac{1}{5}y^{5/2} \right]_4^9 + \left[\tfrac{2}{5}y^{5/2} \right]_0^4 \right]$$

$$= 2\pi \left[\left[\left(-\frac{243}{2} + \frac{243}{2} + \frac{243}{5}\right) - \left(\frac{32}{3} + 24 + \frac{32}{5}\right) \right] + \left[\frac{64}{5} \right] \right] = \boxed{\frac{250}{3}\pi}$$

$\boxed{15}$

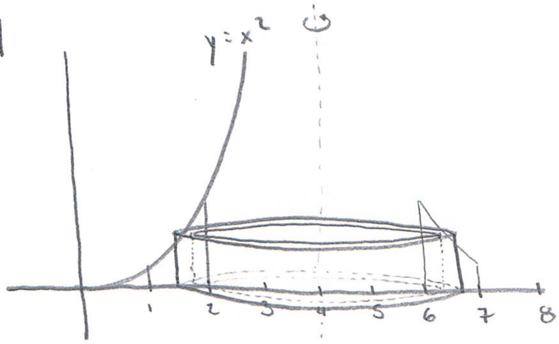


$$V = \int_1^2 2\pi \overset{r}{\overset{\downarrow}{(x-1)}}(x^2)\,dx = 2\pi \int_1^2 x^3 - x^2\,dx$$

$$= 2\pi \left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_1^2$$

$$= 2\pi \left[\left(4 - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right] = 2\pi \left[\frac{4}{3} + \frac{1}{12}\right] = \boxed{\frac{17\pi}{6}}$$

$\boxed{17}$

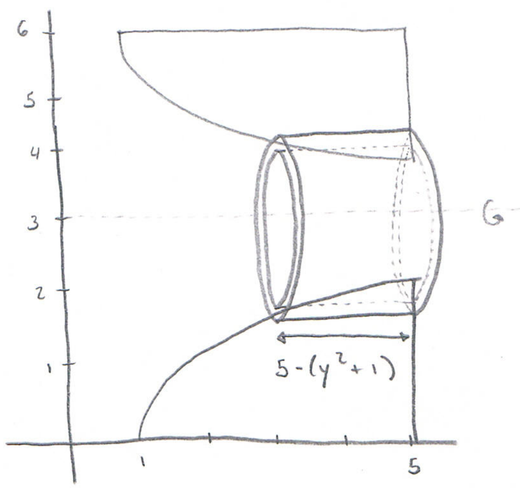


$$V = \int_1^2 2\pi \overset{r}{\overset{\downarrow}{(4-x)}}(x^2)\,dx = 2\pi \int_1^2 4x^2 - x^3\,dx$$

$$= 2\pi \left[\frac{4}{3}x^3 - \frac{1}{4}x^4\right]_1^2 = 2\pi \left[\left(\frac{32}{3} - 4\right) - \left(\frac{4}{3} - \frac{1}{4}\right)\right]$$

$$= 2\pi \left(\frac{28}{3} - \frac{15}{4}\right) = 2\pi \left(\frac{112 - 45}{12}\right) = \boxed{\frac{67\pi}{6}}$$

$\boxed{19}$



$y = \sqrt{x-1} \iff x = y^2 + 1$

$1 \le x \le 5 \qquad 0 \le y \le 2$

$$V = \int_0^2 2\pi (3-y)(5-y^2-1)\,dy$$

$$= 2\pi \int_0^2 y^3 - 3y^2 - 4y + 12\,dy$$

$$= 2\pi \left[\frac{1}{4}y^4 - y^3 - 2y^2 + 12y\right]_0^2 = 2\pi (4 - 8 - 8 + 24) = \boxed{24\pi}$$

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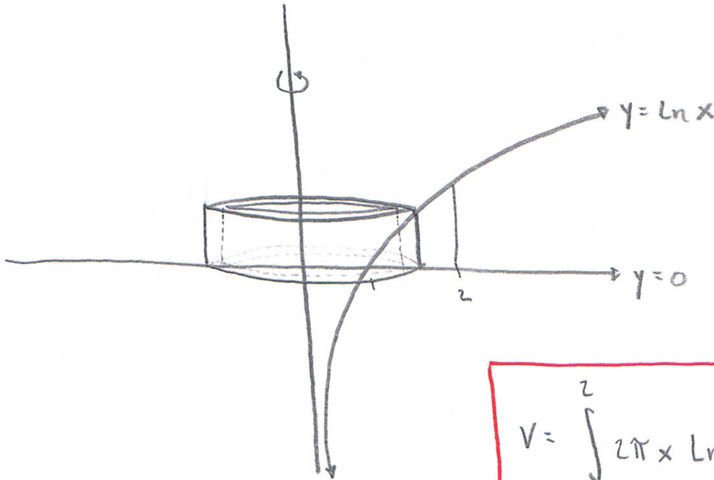

$$\boxed{V = \int_1^2 2\pi x \ln x \, dx}$$

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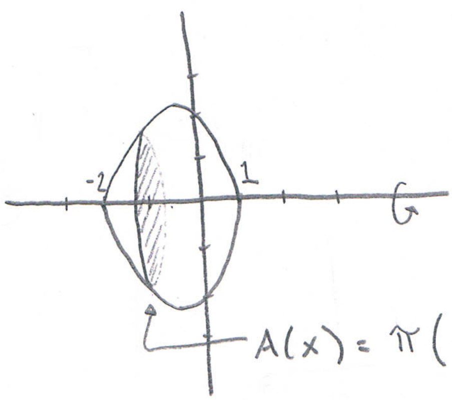
DISK METHOD:



$y = x^2 + x - 2$
$y = 0$
} INTERSECT WHERE

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \, , \; x = 1$

$$A(x) = \pi\left(x^2 + x - 2\right)^2 = \pi\left(x^4 + 2x^3 - 3x^2 - 4x + 4\right)$$

$$V = \int_{-2}^{1} A(x)\,dx = \int_{-2}^{1} \pi\left(x^4 + 2x^3 - 3x^2 - 4x + 4\right) dx$$

$$= \pi\left[\tfrac{1}{5}x^5 + \tfrac{1}{2}x^4 - x^3 - 2x^2 + 4x\right]_{-2}^{1}$$

$$= \pi\left[\left(\tfrac{1}{5} + \tfrac{1}{2} - 1 - 2 + 4\right) - \left(-\tfrac{32}{5} + 8 + 8 - 8 - 8\right)\right]$$

$$= \pi\left[\tfrac{17}{10} + \tfrac{32}{5}\right] = \boxed{\frac{81\pi}{10}}$$

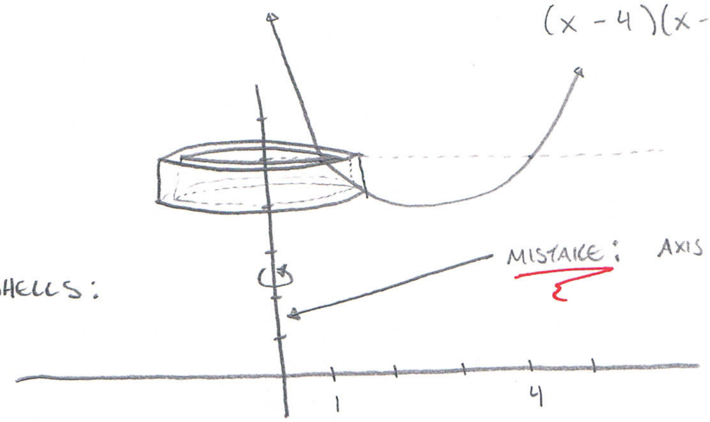
$\boxed{35}$ $\left.\begin{array}{l} y = x + \frac{4}{x} \\ y = 5 \end{array}\right\}$ INTERSECT WHERE $x + \frac{4}{x} = 5$

$$x - 5 + \frac{4}{x} = 0$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0 \longrightarrow x = 1, 4$$

SHELLS:



MISTAKE: AXIS OF ROTATION SHOULD BE $x = -1$ NOT THE y-AXIS

$$V = \int_1^4 2\pi(x+1)\left(5 - x - \frac{4}{x}\right)dx = 2\pi\int_1^4 -x^2 + 4x + 1 - \frac{4}{x}\,dx$$

AGAIN, BECAUSE r IS NOT THE DISTANCE TO THE y-AXIS, BUT RATHER THE DISTANCE TO THE LINE $x = -1$.

$$= 2\pi\left[-\frac{1}{3}x^3 + 2x^2 + x - 4\ln|x|\right]_1^4$$

$$= 2\pi\left[\left(-\frac{64}{3} + 32 + 4 - 4\ln(4)\right) - \left(-\frac{1}{3} + 2 + 1\right)\right]$$

$$= 2\pi\left[-21 + 33 - 4\ln(4)\right] = 2\pi(12 - 4\ln(4)) = \boxed{8\pi(3 - \ln(4))}$$

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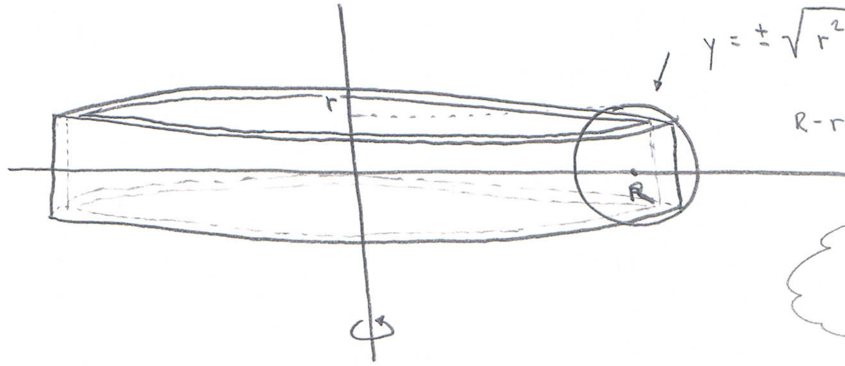

$y = \pm\sqrt{r^2 - (x-R)^2}$

$R - r \le x \le R + r$

VOLUME OF A TORUS

CYLINDRICAL SHELLS: CIRCUMFERENCE $= 2\pi x$

HEIGHT $= 2\sqrt{r^2 - (x-R)^2}$

$$V = \int_{R-r}^{R+r} 4\pi x \sqrt{r^2 - (x-R)^2}\, dx$$

LET $u = x - R \quad \therefore x = u + R$

$du = dx$

$x = R - r \Rightarrow u = -r$

$x = R + r \Rightarrow u = r$

$$\longrightarrow 4\pi \int_{-r}^{r} (u+R)\sqrt{r^2 - u^2}\, du = 4\pi \Bigg[\underbrace{\int_{-r}^{r} u\sqrt{r^2-u^2}\, du}_{(i)} + R \underbrace{\int_{-r}^{r} \sqrt{r^2-u^2}\, du}_{(ii)} \Bigg]$$

(i) LET $v = r^2 - u^2$

$dv = -2u\, du$

$$\longrightarrow \int -\frac{1}{2}\sqrt{v}\, dv = -\frac{1}{3} v^{3/2} \longrightarrow -\frac{1}{3}(r^2-u^2)\Big|_{-r}^{r} = 0$$

(ii) $$\int_{-r}^{r} \sqrt{r^2-u^2}\, du$$

OBSERVE THAT THIS IS HALF THE AREA OF A CIRCLE OF RADIUS r, i.e. $\frac{1}{2}\pi r^2$

NEVERTHELESS, I WILL "USE CALCULUS"...

LET $u = r\sin\theta$

$du = r\cos\theta\, d\theta$

$$\longrightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{r^2 - r^2\sin^2\theta}\; r\cos\theta\, d\theta$$

$$= r^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta = \frac{r^2}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + \cos(2\theta)\, d\theta = \frac{r^2}{2}\Big[\theta + \sin(2\theta)\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi r^2}{2}$$

$$\therefore V = 4\pi R\,(ii) = \boxed{2\pi^2 r^2 R \text{ or } (2\pi R)(\pi r^2)}$$

CUTE!!