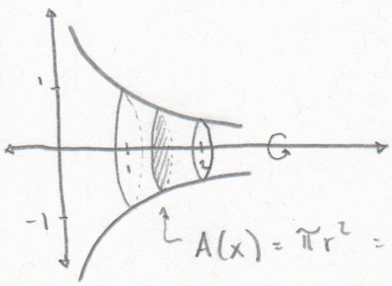
MATH 202

JOHN ADAMSKI
7/2/2012

§7.2

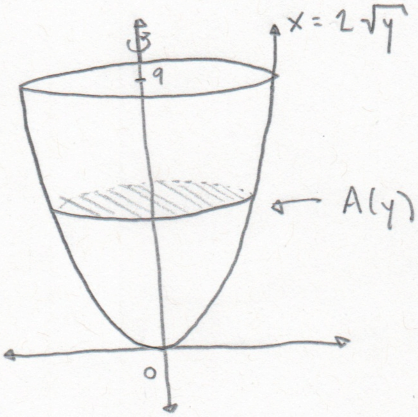
1



$A(x) = \pi r^2 = \pi\left(\frac{1}{x}\right)^2$

$$V = \int_1^2 \pi\left(\frac{1}{x}\right)^2 dx = -\pi x^{-1}\Big|_1^2 = \pi\left(1 - \frac{1}{2}\right) = \boxed{\frac{\pi}{2}}$$

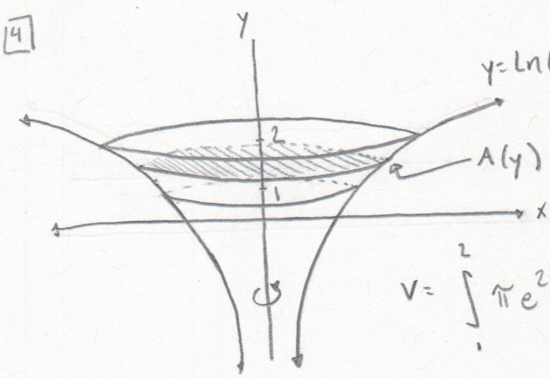
3



$A(y) = \pi r^2 = \pi(2\sqrt{y})^2 = 4\pi y$

$$V = \int_0^9 4\pi y \, dy = 2\pi y^2\Big|_0^9 = \boxed{162\pi}$$

4

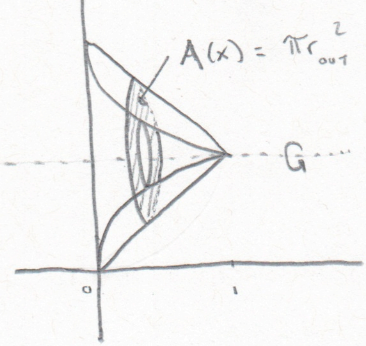


$y = \ln(x) \iff x = e^y$

$A(y) = \pi r^2 = \pi(e^y)^2$

$$V = \int_1^2 \pi e^{2y} dy = \frac{\pi}{2} e^{2y}\Big|_1^2 = \frac{\pi}{2}(e^4 - e^2) = \boxed{\frac{\pi e^2}{2}(e^2 - 1)}$$

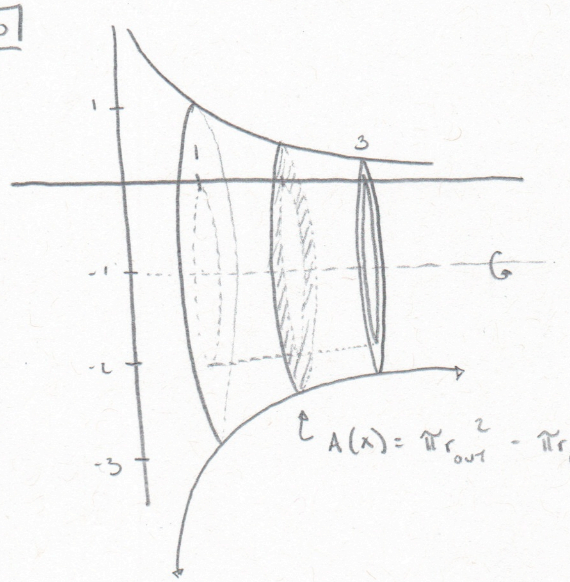
9



$$A(x) = \pi r_{out}^2 - \pi r_{in}^2 = \pi(1-x)^2 - \pi(1-\sqrt{x})^2$$
$$= \pi(1 - 2x + x^2) - \pi(1 - 2\sqrt{x} + x)$$

$$V = \int_0^1 \pi(1 - 2x + x^2) - \pi(1 - 2\sqrt{x} + x)\, dx$$
$$= \pi\int -3x + x^2 + 2\sqrt{x}\, dx = \pi\left(-\frac{3}{2}x^2 + \frac{1}{3}x^3 + \frac{4}{3}x^{3/2}\right)\Big|_0^1 = \boxed{\frac{\pi}{6}}$$

10



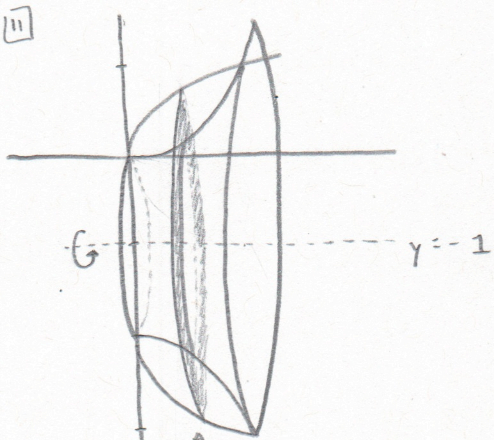
$$A(x) = \pi r_{out}^2 - \pi r_{in}^2 = \pi\left(1 + \frac{1}{x}\right) - \pi(1)^2$$

$$= \pi\left(1 + \frac{2}{x} + \frac{1}{x^2}\right) - \pi = \pi\left(\frac{2}{x} + \frac{1}{x^2}\right)$$

$$V = \int_1^3 A(x)\,dx = \int_1^3 \pi\left(\frac{2}{x} + \frac{1}{x^2}\right)dx = \pi\left(2\ln|x| - \frac{1}{x}\right)\Big|_1^3$$

$$= \pi\left[\left(2\ln(3) - \frac{1}{3}\right) - (0 - 1)\right] = \boxed{2\pi\left(\ln(3) + \frac{1}{3}\right)}$$

11



$$A(x) = \pi r_{out}^2 - \pi r_{in}^2 = \pi(1 + \sqrt{x})^2 - \pi(1 + x^2)^2$$

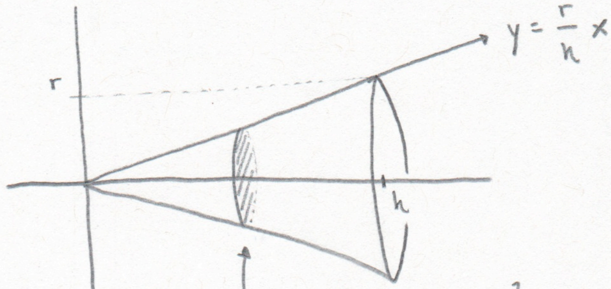
$$= \pi(1 + 2\sqrt{x} + x) - \pi(1 + 2x^2 + x^4)$$

$$= \pi(-x^4 - 2x^2 + x + 2\sqrt{x})$$

$$V = \int_0^1 A(x)\,dx = \pi\int_0^1 -x^4 - 2x^2 + x + 2\sqrt{x}\,dx = \pi\left(-\frac{1}{5}x^5 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + \frac{4}{3}x^{3/2}\right)\Big|_0^1$$

$$= \pi\left(\frac{2}{3} + \frac{1}{2} - \frac{1}{5}\right) = \pi\left(\frac{20 + 15 - 6}{30}\right) = \boxed{\frac{29\pi}{30}}$$

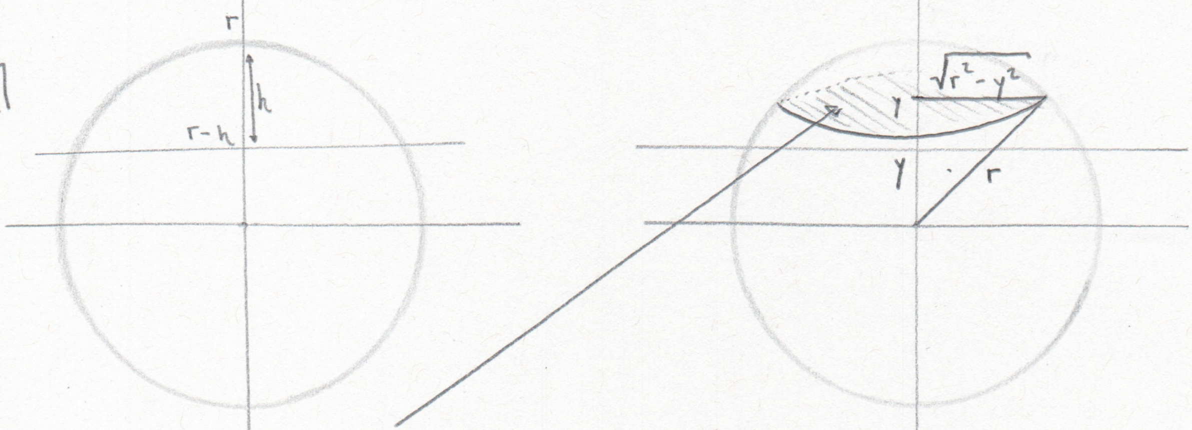
25



$A(x) = \pi\left(\frac{r}{h}x\right)^2 = \frac{\pi r^2}{h^2}x^2$

$$V = \int_0^h \frac{\pi r^2}{h^2}x^2\,dx = \frac{\pi r^2}{3h^2}x^3\Big|_0^h = \boxed{\frac{\pi r^2 h}{3}}$$

27



$A(y) = \pi\left(\sqrt{r^2-y^2}\right)^2$
$= \pi(r^2-y^2)$

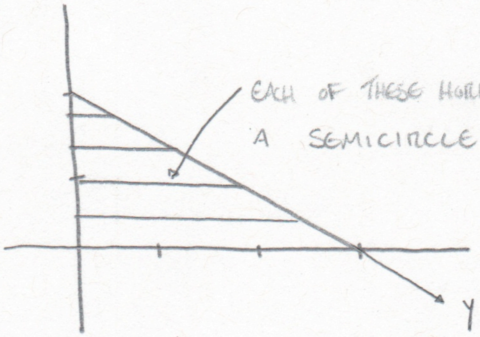
$$V = \int_{r-h}^{r} \pi(r^2-y^2)\,dy = \pi\left(r^2y - \frac{1}{3}y^3\right)\Big|_{r-h}^{r}$$

$$= \pi\left[\left(r^3 - \frac{1}{3}r^3\right) - \left(r^2(r-h) - \frac{1}{3}(r-h)^3\right)\right]$$

$$= \pi\left[r^3 - \frac{1}{3}r^3 \quad - r^3 + r^2h + \frac{1}{3}r^3 - r^2h + rh^2 - \frac{1}{3}h^3\right]$$

$$= \pi\left[rh^2 - \frac{1}{3}h^3\right] = \boxed{\pi h^2\left(r - \frac{1}{3}h\right)}$$

$\boxed{36}$



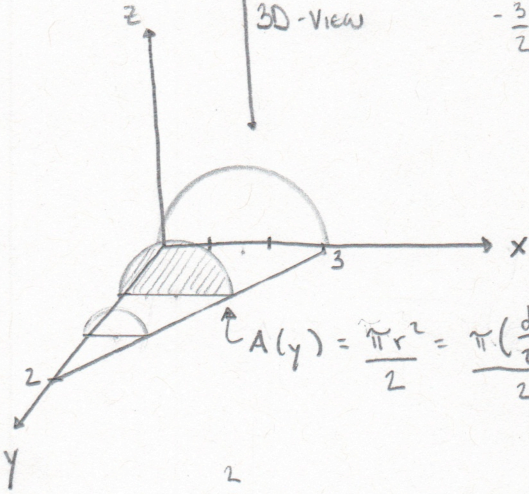
EACH OF THESE HORIZONTAL LINES REPRESENTS A DIAMETER OF A SEMICIRCLE CROSS SECTION

$y = -\frac{2}{3}x + 2$

$3y = -2x + 6$

$-\frac{3}{2}y + 3 = x$

3D-VIEW

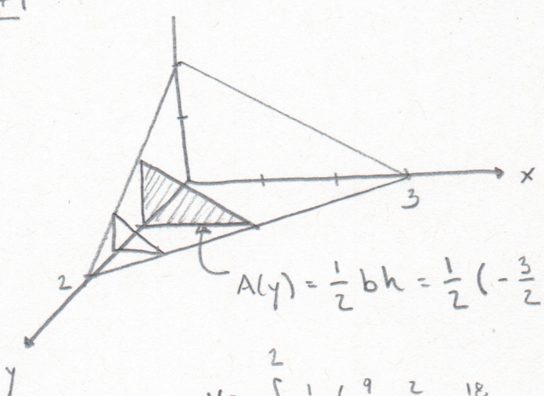


$A(y) = \frac{\pi r^2}{2} = \frac{\pi\left(\frac{d}{2}\right)^2}{2} = \frac{\pi}{8}\left(-\frac{3}{2}y + 3\right)^2$

$$V = \int_0^2 \frac{\pi}{8}\left(-\frac{3}{2}y + 3\right)^2 dy = \frac{\pi}{8}\int_0^2 \frac{9}{4}y^2 - \frac{18}{2}y + 9\, dy$$

$$= \frac{\pi}{8}\left(\frac{9}{12}y^3 - \frac{18}{4}y^2 + 9y\right)\Big|_0^2 = \frac{\pi}{8}\left(\frac{72}{12} - \frac{72}{4} + 18\right) = \frac{\pi}{8}(6 - 18 + 18) = \boxed{\frac{3}{4}\pi}$$

$\boxed{37}$



$A(y) = \frac{1}{2}bh = \frac{1}{2}\left(-\frac{3}{2}y + 3\right)^2 = \frac{1}{2}\left(\frac{9}{4}y^2 - \frac{18}{2}y + 9\right)$

$$V = \int_0^2 \frac{1}{2}\left(\frac{9}{4}y^2 - \frac{18}{2}y + 9\right) dy = \frac{1}{2}\left(\frac{3}{4}y^3 - \frac{9}{2}y^2 + 9y\right)\Big|_0^2$$

$$= \frac{1}{2}(6 - 18 + 18) = \boxed{3}$$

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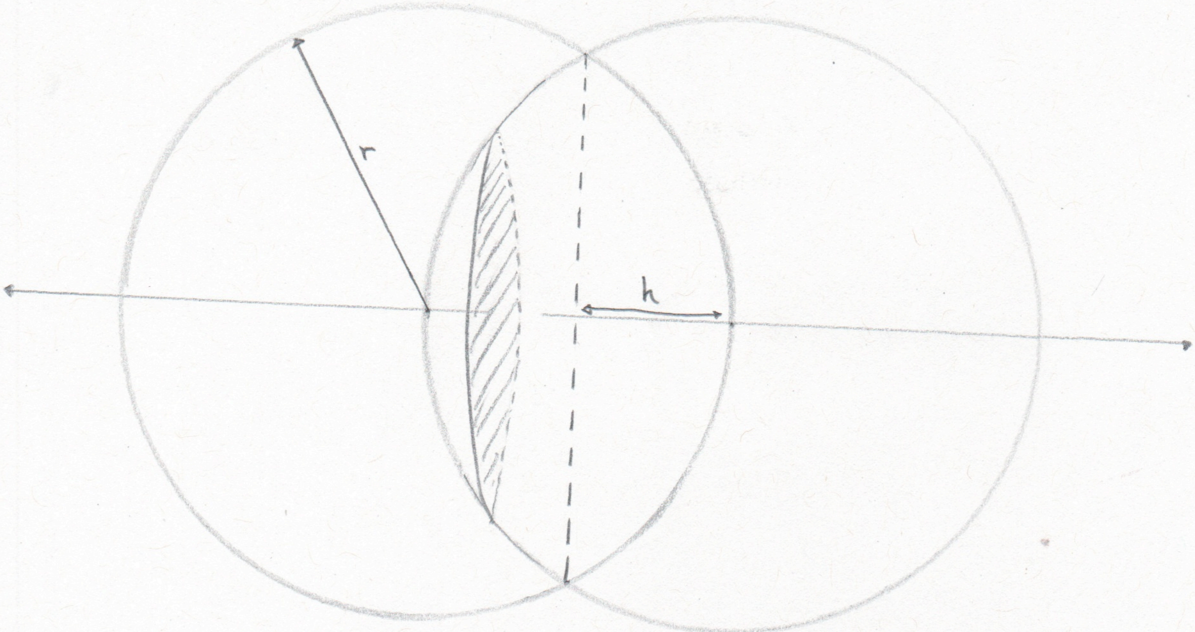

NOTE THAT THE SOLID IS COMPOSED OF TWO CAPS OF SPHERES OF HEIGHT $h = \frac{r}{2}$ (BY SYMMETRY). [SEE EXERCISE # 27]

$$V = 2\int_{r-h}^{r} \pi (r^2 - x^2)\,dx = 2\pi h^2 \left(r - \tfrac{1}{3}h\right) \quad \text{WHERE } h = \frac{r}{2}$$

$$= 2\pi \left(\frac{r}{2}\right)^2 \left(r - \frac{1}{3}\left(\frac{r}{2}\right)\right)$$

$$= \frac{\pi r^2}{2}\left(\frac{5}{6} r\right) = \boxed{\frac{5\pi r^3}{12}}$$