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MATH 202

§7.1

1. $y_T = 5x - x^2$; $y_B = x$

THE POINTS OF INTERSECTION ARE $(0,0)$, AND $(4,4)$ [GIVEN]

$$\therefore \text{AREA} = \int_0^4 (y_T - y_B)\,dx = \int_0^4 5x - x^2 - x\,dx = \int_0^4 4x - x^2\,dx$$

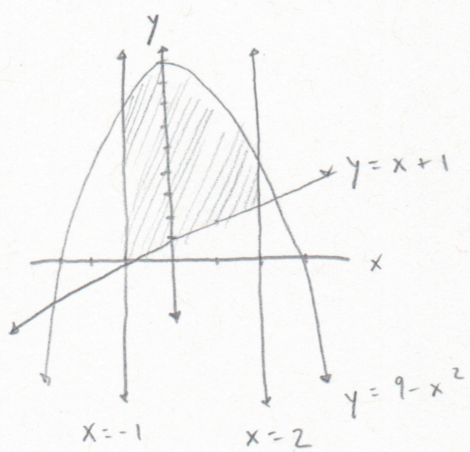
$$= 2x^2 - \frac{1}{3}x^3 \Big|_0^4 = \left(32 - \frac{64}{3}\right) - (0) = \boxed{\frac{32}{3}}$$

3. THIS TIME, WE INTEGRATE VERTICALLY, FROM $y = -1$, TO $y = 1$.

$$\text{AREA} = \int_{-1}^{1} (x_R - x_L)\,dy = \int_{-1}^{1} e^y - (y^2 - 2)\,dy = e^y - \frac{1}{3}y^3 + 2y \Big|_{-1}^{1}$$

$$= \left(e - \frac{1}{3} + 2\right) - \left(e^{-1} + \frac{1}{3} - 2\right) = \boxed{\frac{10}{3} + e - \frac{1}{e}}$$

5.



$y_T = 9 - x^2$; $y_B = x + 1$

$$\text{AREA} = \int_{-1}^{2} (9 - x^2) - (x + 1)\,dx$$

$$= \int_{-1}^{2} 8 - x - x^2\,dx = 8x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_{-1}^{2}$$

$$= \left(16 - 2 - \frac{8}{3}\right) - \left(-8 - \frac{1}{2} + \frac{1}{3}\right) = 24 - \frac{3}{2} - 3 = \boxed{19\tfrac{1}{2}}$$

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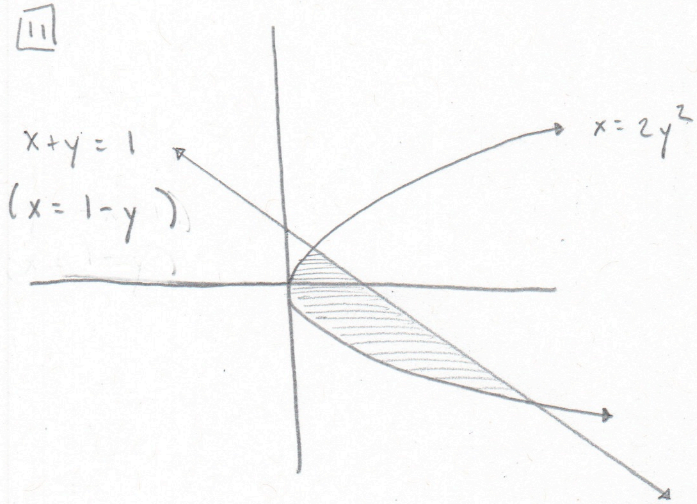

INTERSECTION: $1-y=2y^2$

$0=2y^2+y-1=(2y-1)(y+1)$

$y=\frac{1}{2}, -1$

$\therefore$ AREA $= \int_{-1}^{1/2} (x_R - x_L)\,dy$

$= \int_{-1}^{1/2} (1-y)-(2y^2)\,dy = y - \frac{1}{2}y^2 - \frac{2}{3}y^3 \Big|_{-1}^{1/2} = \left(\frac{1}{2}-\frac{1}{8}-\frac{1}{12}\right)-\left(-1-\frac{1}{2}+\frac{2}{3}\right)$

$= \frac{7}{24}+\frac{5}{6} = \frac{27}{24} = \boxed{\frac{9}{8}}$

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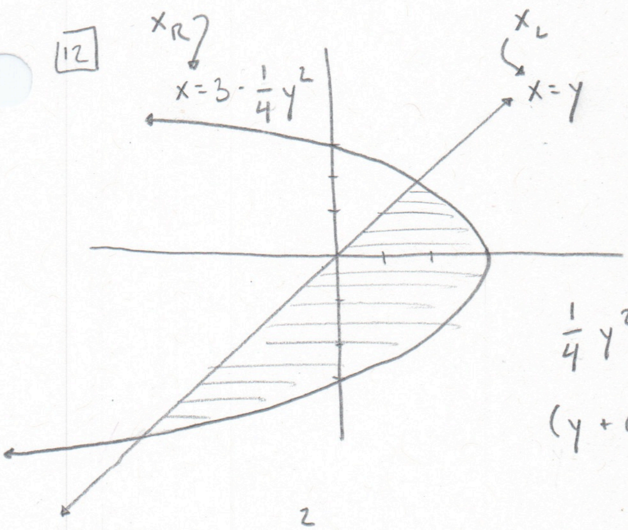

$4x+y^2=12$

$x=3-\frac{1}{4}y^2$

INTERSECTIONS: $y=3-\frac{1}{4}y^2$

$\frac{1}{4}y^2+y-3=0 \rightarrow y^2+4y-12=0$

$(y+6)(y-2)=0 \longrightarrow y=-6, 2$

$\therefore$ AREA $= \int_{-6}^{2} \left(3-\frac{1}{4}y^2\right)-(y)\,dy = 3y - \frac{1}{12}y^3 - \frac{1}{2}y^2 \Big|_{-6}^{2}$

$= \left(6-\frac{2}{3}-2\right)-(-18+18-18) = \boxed{-14\frac{2}{3} \text{ or } -\frac{44}{3}}$

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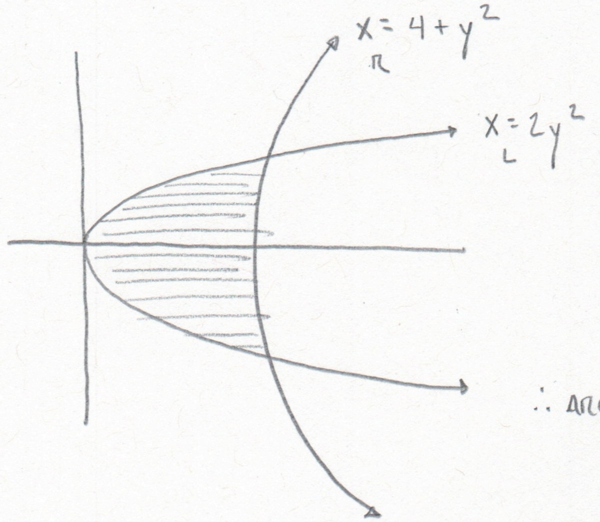


INTERSECTION POINTS:

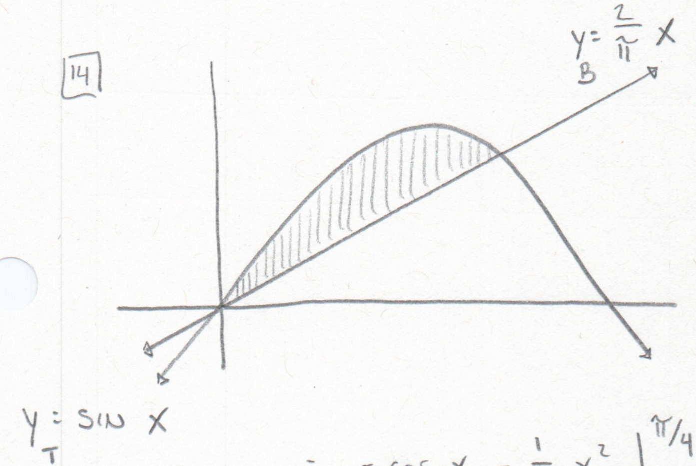
$$4+y^2 = 2y^2 \rightarrow 0 = y^2-4 = (y+2)(y-2)$$

$$y = \pm 2$$

$$\therefore \text{AREA} = \int_{-2}^{2} x_R - x_L \, dy = \int_{-2}^{2} 4-y^2 \, dy$$

$$= 4y - \frac{1}{3}y^3 \Big|_{-2}^{2} = \left(8-\frac{8}{3}\right) - \left(-8+\frac{8}{3}\right) = 16 - \frac{16}{3} = \boxed{\frac{32}{3}}$$

14



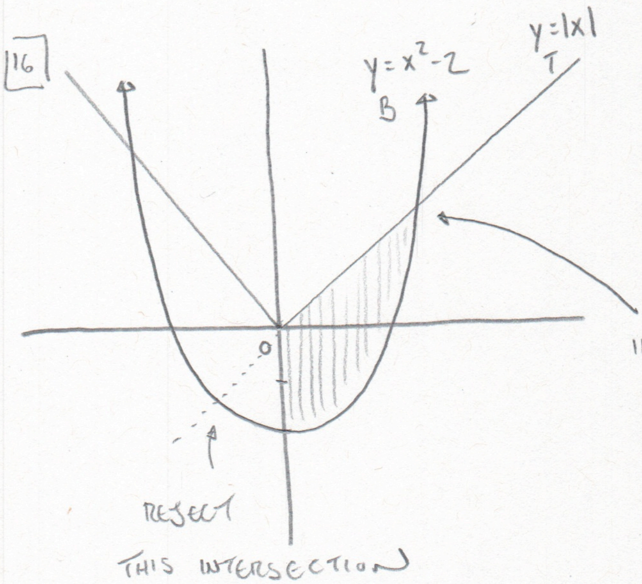
INTERSECTION: $x = 0, \frac{\pi}{4}$

I GOT THIS BY "GUESS & CHECK" :-/

$$\therefore \text{AREA} = \int_{0}^{\pi/4} \sin x - \frac{2}{\pi}x \, dx$$

$$= -\cos x - \frac{1}{\pi}x^2 \Big|_{0}^{\pi/4} = \left(-\frac{\sqrt{2}}{2} - \frac{\pi}{16}\right) - (-1) = \boxed{1 - \frac{\sqrt{2}}{2} - \frac{\pi}{16}}$$

16



NOTE: BY SYMMETRY (BOTH $|x|$ & x^2-2 ARE EVEN FUNCTIONS) IT IS ENOUGH TO FIND AREA OF RIGHT HALF ($x \geq 0$) AND DOUBLE.

INTERSECTION: $x = x^2-2 \rightarrow x^2-x-2=0$

$$(x-2)(x+1) = 0$$

$$x = 2 \qquad x = -1$$

REJECT!

$$\therefore \text{AREA} = 2\int_0^2 x - (x^2 - 2)\,dx = 2\left[\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\right]_0^2$$

$$= 2\left[2 - \frac{8}{3} + 4\right] = \boxed{\frac{16}{3}}$$

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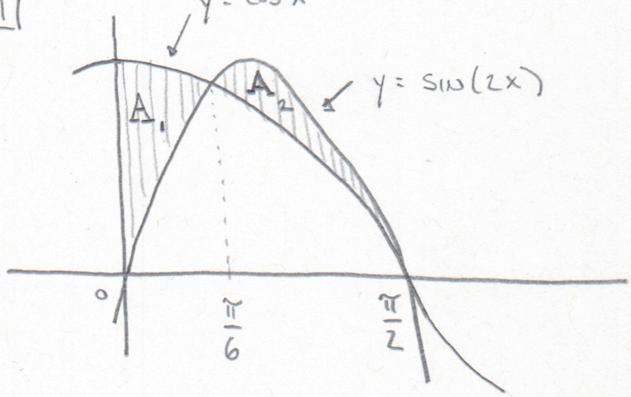

THIS QUESTION IS DIFFERENT BECAUSE THE ROLES OF Y_T & Y_B SWITCH WHEN $\cos x = \sin 2x$

i.e. WHEN $x = \frac{\pi}{6}$ ("GUESS & CHECK")

$$\therefore A_1 = \int_0^{\pi/6} \cos x - \sin(2x)\,dx = \sin x + \frac{1}{2}\cos(2x)\Big|_0^{\pi/6} = \left(\frac{1}{2} + \frac{1}{4}\right) - \left(0 + \frac{1}{2}\right) = \frac{1}{4}$$

$$A_2 = \int_{\pi/6}^{\pi/2} \sin(2x) - \cos x\,dx = -\frac{1}{2}\cos 2x - \sin x\Big|_{\pi/6}^{\pi/2} = \left(\frac{1}{2} - 1\right) - \left(-\frac{1}{4} - \frac{1}{2}\right) = \frac{1}{4}$$

$$\therefore A = A_1 + A_2 = \boxed{\frac{1}{4}}$$

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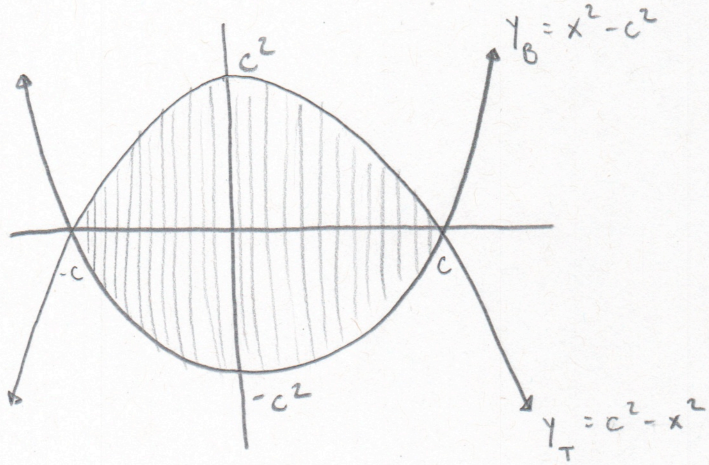


THESE CURVES INTERSECT WHEN $y = 0$,

i.e. WHEN $x^2 - c^2 = 0$

i.e. WHEN $x = \pm c$.

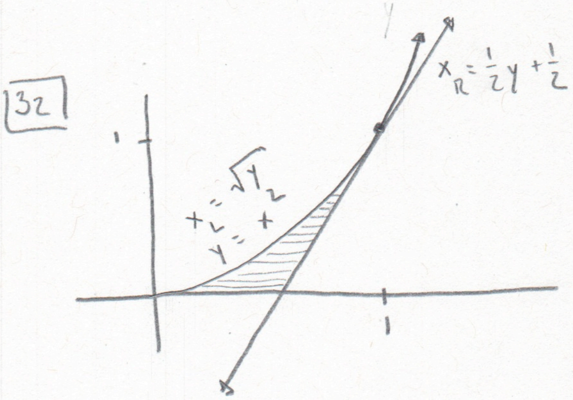
$$\therefore A = \int_{-c}^{c} (c^2 - x^2) - (x^2 - c^2)\, dx = 2c^2 x - \frac{2}{3} x^3 \Big|_{-c}^{c}$$

$$= \left(2c^3 - \frac{2}{3}c^3\right) - \left(-2c^3 + \frac{2}{3}c^3\right) = \frac{8}{3}c^3$$

THEN $\frac{8}{3}c^3 = 576 \rightarrow c^3 = 216 \rightarrow \boxed{c = 6}$

(ALTHOUGH $c = -6$ WOULD WORK TOO :))

[32]



TANGENT LINE AT (1,1) TO $y = f(x) = x^2$

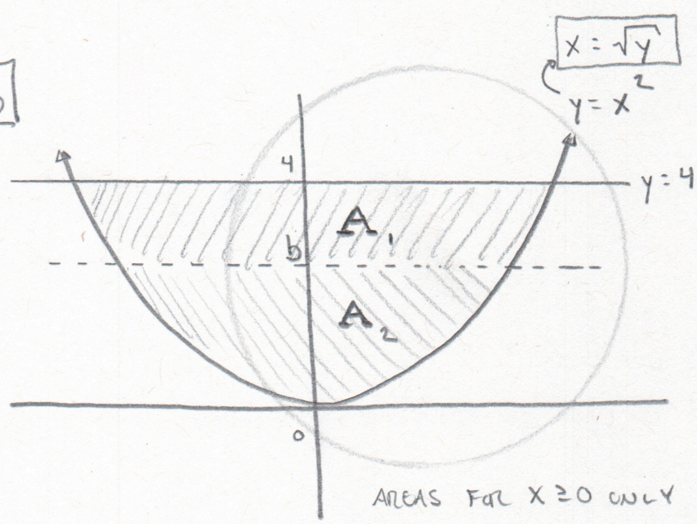
$$y - 1 = f'(1)(x-1) = 2(x-1)$$

$$y - 1 = 2x - 2 \rightarrow x = \frac{1}{2}y + \frac{1}{2}$$

$$\therefore \text{AREA} = \int_0^1 x_R - x_L \, dy = \int_0^1 \left(\frac{1}{2}y + \frac{1}{2}\right) - \sqrt{y}\, dy = \frac{1}{4}y^2 + \frac{1}{2}y - \frac{2}{3}y^{3/2} \Big|_0^1$$

$$= \left(\frac{1}{4} + \frac{1}{2} - \frac{2}{3}\right) - 0 = \boxed{\frac{1}{12}}$$

[33]



AREAS FOR $X \geq 0$ ONLY

NOTE: BY SYMMETRY ($y = 4$ & $y = x^2$ ARE BOTH EVEN FUNCTIONS), IT IS ENOUGH TO WORK WITH $X \geq 0$ ONLY.

$$A_1 = \int_b^4 \sqrt{y}\, dy = \frac{2}{3} y^{3/2} \Big|_b^4 = \frac{2}{3}\left(8 - b^{3/2}\right) = \frac{16}{3} - \frac{2b^{3/2}}{3}$$

$$A_2 = \int_0^b \sqrt{y}\, dy = \frac{2}{3} y^{3/2} \Big|_0^b = \frac{2}{3}\left(b^{3/2} - 0\right) = \frac{2b^{3/2}}{3}$$

SET $A_1 = A_2 \longrightarrow \frac{16}{3} = \frac{4}{3} b^{3/2} \longrightarrow 4 = b^{3/2} \longrightarrow \boxed{b = 4^{2/3}}$