MATH 202

JOHN ADAMSKI
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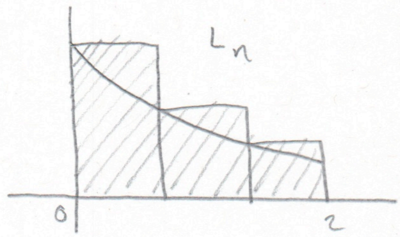
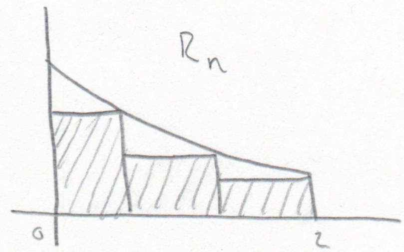
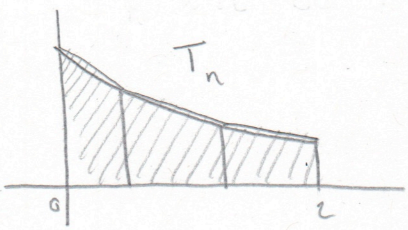
§ 6.5

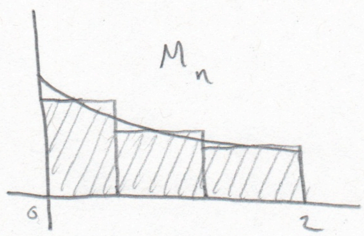
2. IT IS CLEAR THAT L_n WILL BE THE GREATEST OVERESTIMATE WHILE R_n WILL BE THE LOWEST UNDERESTIMATE.





FURTHERMORE T_n WILL BE AN OVERESTIMATE & M_n WILL BE AN UNDERESTIMATE (THOUGH LESS EXTREME THAN L_n & R_n, RESPECTIVELY.





$\therefore$ WE HAVE $R_n \le M_n \le T_n \le L_n$

(a) $R_n = .7811$, $M_n = .8632$, $T_n = .8675$, $L_n = .9540$

(b) $M_n = .8632 \le \int_0^2 f(x)\,dx \le .8675 = T_n$

1. $\Delta x = \dfrac{1-0}{4} = \dfrac{1}{4} \Rightarrow x_0 = 0,\ x_1 = \frac{1}{4},\ x_2 = \frac{1}{2},\ x_3 = \frac{3}{4},\ x_4 = 1$

$\bar{x}_1 = \frac{1}{8}$ $\quad \bar{x}_2 = \frac{3}{8}$ $\quad \bar{x}_3 = \frac{5}{8}$ $\quad \bar{x}_4 = \frac{7}{8}$

$\therefore M_4 = \sum_{i=1}^{4} f(\bar{x}_i)\Delta x = \frac{1}{4}\left[\cos\left(\frac{1}{64}\right) + \cos\left(\frac{9}{64}\right) + \cos\left(\frac{25}{64}\right) + \cos\left(\frac{49}{64}\right)\right]$

$\approx \boxed{.9089}$

$$T_4 = \frac{\Delta x}{2}\left(f(0) + 2f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 2f(\tfrac{3}{4}) + f(1) \right)$$

$$= \frac{1}{8}\left(\cos(0) + 2\cos(\tfrac{1}{16}) + 2\cos(\tfrac{1}{4}) + 2\cos(\tfrac{9}{16}) + \cos(1) \right)$$

$$\approx \boxed{.8958}$$

SINCE THE GRAPH IS CONCAVE DOWN, T_4 IS AN UNDERESTIMATE & M_4 IS AN OVERESTIMATE.

$\boxed{17}$ $f(x) = e^{-x^2}$

$$f'(x) = -2x e^{-x^2}$$

$$f''(x) = -2\left(e^{-x^2} - 2x^2 e^{-x^2}\right) = 2e^{-x^2}(2x^2 - 1)$$

← WHAT IS MAX ABS. VAL. FOR $0 \le x \le 2$?

$$f'''(x) = -4xe^{-x^2}(2x^2-1) + 2e^{-x^2}(4x) = 0$$

$$= 4xe^{-x^2}(3 - 2x^2) = 0 \implies 3 - 2x^2 = 0$$

$$\implies x = \sqrt{\frac{3}{2}}$$

← SO WE WILL CHECK $|f''(x)|$ HERE & AT ENDPOINTS.

$$|f''(\sqrt{\tfrac{3}{2}})| = |2e^{-3/2}(3-1)| = 4e^{-3/2} \approx .8925$$

$$|f''(0)| = |2e^{0}(0-1)| = 2$$ ← LET $K = 2$.

$$|f''(1)| = |2e^{-1}(2-1)| = \frac{2}{e} < 1$$

THEN $$|E_T| \le \frac{2 \cdot 2^3}{12n^2} < \frac{1}{100{,}000} \implies n^2 > \frac{1{,}600{,}000}{12}$$

$$\implies n > 365.1 \implies \boxed{n = 366 \text{ FOR } T_n}$$

$$|E_M| \le \frac{2 \cdot 2^3}{24n^2} < \frac{1}{100{,}000} \implies n^2 > \frac{1{,}600{,}000}{24}$$

$$\implies n > 258.2 \implies \boxed{n = 259 \text{ FOR } M_n}$$

18. $f(x) = \cos(x^2)$

$$f'(x) = -\sin(x^2) \cdot 2x = -2x\sin(x^2)$$

$$f''(x) = -2\sin(x^2) - 2x\cos(x^2) \cdot 2x$$

$$= -2\left[\sin(x^2) + 2x\cos(x^2)\right]$$

$$\left.\begin{aligned} f'''(x) &= -2\left[2x\cos(x^2) + 2\cos(x^2) - 4x^2\sin(x^2)\right] \\ &= -4\left[(x+1)\cos(x^2) - 2x^2\sin(x^2)\right] = 0 \\ &(x+1)\cos(x^2) = 2x^2\sin(x^2) \end{aligned}\right\} \text{LOOKING FOR LOCAL MAX/MIN}$$

UMMM... $x = 1$ IS A SOL'N

$$|f''(0)| = \left|-2\left[\sin(0) + 2(0)\cos(0)\right]\right| = 0$$

$$|f''(1)| = \left|-2\left[\sin(1) + 2(1)\cos(1)\right]\right| = 5.5271 \longleftarrow \text{LET THIS BE K}$$

$$\therefore \quad |E_T| \le \frac{5.5271}{12n^2} < \frac{1}{100{,}000} \Rightarrow n > \sqrt{\frac{552710}{12}} \approx 214.6$$

$$\boxed{n = 215 \text{ FOR } T_n}$$

$$|E_M| \le \frac{5.5271}{24n^2} < \frac{1}{100{,}000} \Rightarrow n > \sqrt{\frac{552710}{24}} \approx 151.8$$

$$\boxed{n = 152 \quad M_n}$$

19 $f(x) = e^x$

$f'(x) = f''(x) = e^x$ ALWAYS INCREASING

$\therefore K = f''(1) = e$

$$|E_T| \le \frac{e}{12n^2} < \frac{1}{100{,}000} \Rightarrow n < \sqrt{\frac{271828}{12}} \approx 150.5$$

$$\boxed{n = 151 \text{ FOR } T_n}$$

$$|E_M| \le \frac{e}{24n^2} < \frac{1}{100{,}000} \Rightarrow n < \sqrt{\frac{271828}{24}} \approx 106.4$$

$$\boxed{n = 107 \text{ FOR } M_n}$$

20 PICKING UP WHERE WE LEFT OFF IN # 17 ...

$$f'''(x) = 4xe^{-x^2}(3-2x^2)$$

$$f^{(4)}(x) = 4\left[e^{-x^2}(3-2x^2) + x\left(-2xe^{-x^2}(3-2x^2) + e^{-x^2}(-4x)\right)\right]$$

NOW, WHAT IS THE MAX ABS. VAL. OF THIS FOR $0 \le x \le 1$??

NO IDEA ... BUT THE GRAPH SUGGESTS THIS MAX OCCURS WHEN $x = 0$:/

$$f^{(4)}(0) = 4[1(3) + 0 + 0] = 12 \leftarrow \text{LET THIS BE } K$$

$$\therefore |E_S| \le \frac{12}{180n^4} < \frac{1}{100{,}000} \Rightarrow n > \sqrt[4]{\frac{1{,}200{,}000}{180}} \approx 9.03$$

$$\boxed{n = 10 \text{ FOR } S_n}$$

COMPARE TO $n = 259$ FOR M_n !!!

25 (b) NOTE: $\bar{x}_1 = \frac{1}{2}$, $\bar{x}_2 = \frac{3}{2}$, $\bar{x}_3 = \frac{5}{2}$, $\bar{x}_4 = \frac{7}{2}$

$$\Delta x = \frac{4-0}{4} = 1$$

$$M_4 = 1\left[f\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{2}\right) + f\left(\tfrac{5}{2}\right) + f\left(\tfrac{7}{2}\right)\right]$$

$$= 1\left[1 + 4\tfrac{1}{2} + 4\tfrac{1}{2} + 2\right] = \boxed{12} \text{ (b)}$$

(a) $$T_4 = \frac{1}{2}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right]$$

$$= \frac{1}{2}\left[0 + 2(3) + 2(5) + 2(3) + 1\right] = \boxed{11\tfrac{1}{2}} \text{ (a)}$$

(c) $$S_4 = \frac{1}{3}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right]$$

$$= \frac{1}{3}\left[0 + 4(3) + 2(5) + 4(3) + 1\right] = \boxed{11\tfrac{2}{3}} \text{ (c)}$$

26 NOTE THAT $n = 10$ (EVEN) WITH $\Delta x = 0.5$ SEC

$$S_{10} = \frac{0.5}{3}\Big[v(0) + 4v(.5) + 2v(1) + 4v(1.5) + 2v(2)$$
$$+ 4v(2.5) + 2v(3) + 4v(3.5) + 2v(4)$$
$$+ 4v(4.5) + v(5)\Big]$$

$$= \frac{1}{6}\Big[0 + 4(4.67) + 2(7.34) + 4(8.86) + 2(9.73) + 4(10.22)$$
$$+ 2(10.51) + 4(10.67) + 2(10.76) + 4(10.81) + 10.81\Big]$$

$$= \boxed{44.735 \text{ m}}$$

28

$n = 6$, $\Delta x = \dfrac{6-0}{6} = 1$

$$S_6 = \frac{1}{3}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + 2f(4) + 4f(5) + f(6)\right]$$

$$= \frac{1}{3}\left[4 + 4(3) + 2(2\tfrac{1}{2}) + 4(2) + 2(1\tfrac{1}{2}) + 4(1) + 1\right]$$

$$= \boxed{12\tfrac{1}{3}}$$ ABOUT

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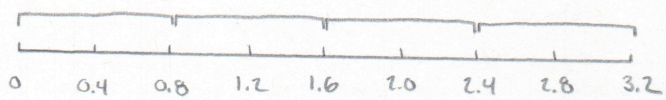

$\bar{x}_1$ $\bar{x}_2$ $\bar{x}_3$ $\bar{x}_4$ ← MIDPOINTS OF THE 4 SUBINTERVALS.

$$\Delta x = \frac{3.2 - 0}{4} = 0.8$$

$$M_4 = 0.8\left[f(0.4) + f(1.2) + f(2.0) + f(2.8)\right]$$

$$= 0.8\left[6.5 + 6.4 + 7.6 + 8.8\right] = \boxed{23.44}$$ (a)

(b) $-4 \le f''(x) \le 1 \Rightarrow |f''(x)| \le 4$ ← LET THIS BE K

$$|E_M| \le \frac{4(3.2)^3}{24(4)^2} = \frac{131.072}{384} \doteq \boxed{.341\overline{3}}$$

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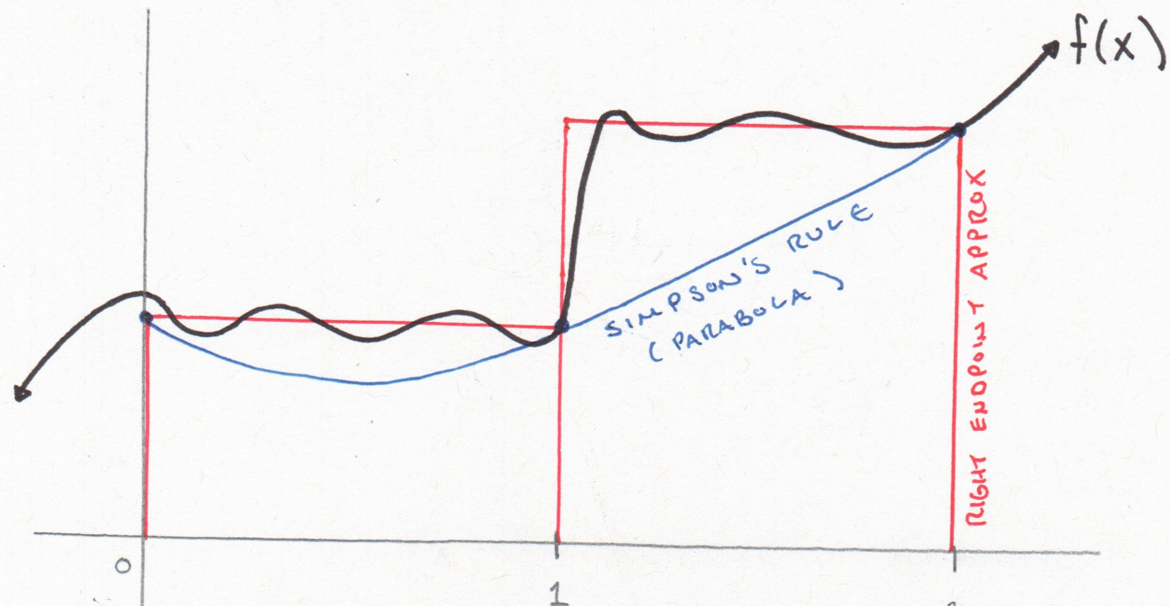
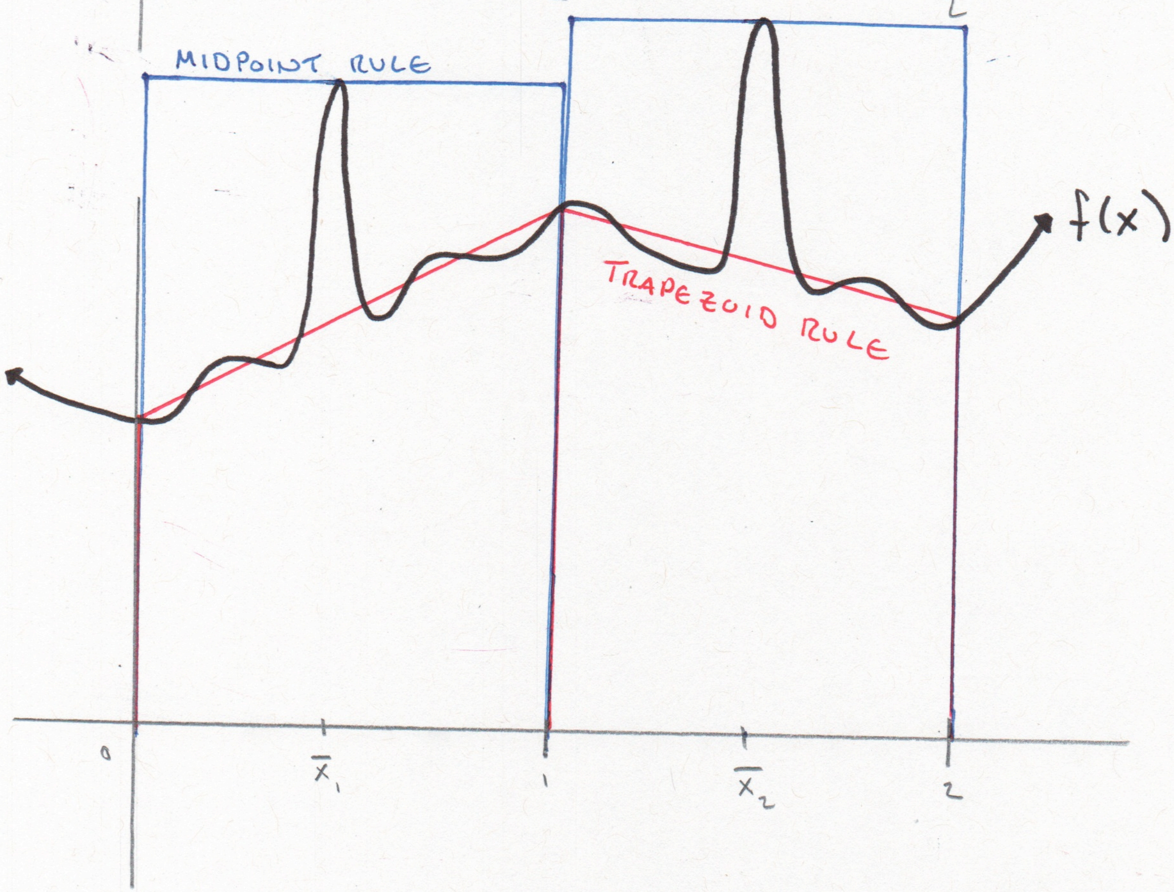

f(x)
SIMPSON'S RULE
(PARABOLA)
RIGHT ENDPOINT APPROX
0
1
2

35


MIDPOINT RULE
f(x)
TRAPEZOID RULE
0
$\bar{x}_1$
1
$\bar{x}_2$
2