MATH 202

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§6.1

LISTENING TO: TALKING HEADS

[3] $\int x\cos(5x)\,dx$

LET $u = x$ $\quad v = \frac{1}{5}\sin(5x)$

$du = dx$ $\quad dv = \cos(5x)\,dx$

$$= \frac{1}{5}x\sin(5x) - \int \frac{1}{5}\sin(5x)\,dx$$

$$= \boxed{\frac{1}{5}\sin(5x) + \frac{1}{25}\cos(5x) + C}$$

[5] $\int re^{r/2}\,dr$

LET $u = r$ $\quad v = 2e^{r/2}$

$du = dr$ $\quad dv = e^{r/2}\,dr$

$$= 2re^{r/2} - \int 2e^{r/2}\,dr$$

$$= \boxed{2re^{r/2} - 4e^{r/2} + C}$$

7 $\int x^2 \sin(\pi x)\,dx$

LET $u = x^2$ $\quad v = -\frac{1}{\pi}\cos(\pi x)$

$du = 2x\,dx$ $\quad dv = \sin(\pi x)\,dx$

$$= -\frac{1}{\pi}x^2\cos(\pi x) + \frac{2}{\pi}\int x\cos(\pi x)\,dx$$

AGAIN! LET $u = x$ $\quad v = \frac{1}{\pi}\sin(\pi x)$

$du = dx$ $\quad dv = \cos(\pi x)\,dx$

$$= -\frac{1}{\pi}x^2\cos(\pi x) + \frac{2}{\pi}\left[\frac{1}{\pi}x\sin(\pi x) - \frac{1}{\pi}\int \sin(\pi x)\,dx\right]$$

$$+ \frac{1}{\pi^2}\cos(\pi x)$$

$$= \boxed{-\frac{1}{\pi}x^2\cos(\pi x) + \frac{2}{\pi^2}x\sin(\pi x) + \frac{2}{\pi^3}\cos(\pi x) + C}$$

9 $\int \ln(2x+1)\,dx$

LET $t = 2x+1$

$dt = 2\,dx \Rightarrow \frac{1}{2}dt = dx$

$\frac{1}{2}\int \ln(t)\,dt$

LET $u = \ln(t)$ $\quad v = t$

$du = \frac{1}{t}\,dt$ $\quad dv = dt$

$$= \frac{1}{2}\left[t\ln(t) - \int \frac{t}{t}\,dt\right] = \frac{1}{2}\left[t\ln(t) - t\right]$$

$$= \frac{1}{2}(2x+1)\ln(2x+1) - \frac{1}{2}(2x+1) + C$$

$$= \boxed{\frac{1}{2}(2x+1)\ln(2x+1) - x + C}$$

$\boxed{11}$ $\int \arctan(4t)\,dt$ LET $u = \arctan(4t)$ $v = t$

$du = \frac{1}{1+16t^2}\,dt$ $dv = dt$

$= t\arctan(4t) - \int \frac{t}{1+16t^2}\,dt$ LET $u = 1+16t^2$

$du = 32t\,dt$

$\frac{1}{32}\int \frac{1}{u}\,du = \frac{1}{32}\ln|u| + C$

$= \boxed{t\arctan(4t) - \frac{1}{32}\ln(1+16t^2) + C}$

$\boxed{13}$ $\int e^{2\theta}\sin(3\theta)\,d\theta$ LET $u = e^{2\theta}$ $v = -\frac{1}{3}\cos(3\theta)$

$du = 2e^{2\theta}\,d\theta$ $dv = \sin(3\theta)\,d\theta$

$= -\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{3}\int e^{2\theta}\cos(3\theta)\,d\theta$

AGAIN! LET $u = e^{2\theta}$ $v = \frac{1}{3}\sin(3\theta)$

$du = 2e^{2\theta}\,d\theta$ $dv = \cos(3\theta)\,d\theta$

$= -\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{3}\left[\frac{1}{3}e^{2\theta}\sin(3\theta) - \frac{2}{3}\int e^{2\theta}\sin(3\theta)\,d\theta\right]$

$\therefore \int e^{2\theta}\sin(3\theta)\,d\theta = -\frac{1}{3}e^{2\theta}\cos(3\theta) + \frac{2}{9}e^{2\theta}\sin(3\theta) - \frac{4}{9}\int e^{2\theta}\sin(3\theta)\,d\theta$

$\Rightarrow \frac{13}{9}\int e^{2\theta}\sin(3\theta)\,d\theta = e^{2\theta}\left(\frac{2}{9}\sin(3\theta) - \frac{1}{3}\cos(3\theta)\right)$

$\int e^{2\theta}\sin(3\theta)\,d\theta = \boxed{\frac{1}{13}e^{2\theta}\left(2\sin(3\theta) - 3\cos(3\theta)\right)}$

15 $\int_0^{\pi} t \sin(3t)\, dt$ LET $u = t$ $v = -\frac{1}{3}\cos(3t)$

$du = dt$ $dv = \sin(3t)\, dt$

$$= -\frac{1}{3} t \cos(3t) \Big|_0^{\pi} + \frac{1}{3}\int_0^{\pi} \cos(3t)\, dt$$

$$= \frac{\pi}{3} + \left[\frac{1}{9}\sin(3t)\right]_0^{\pi} = \boxed{\frac{\pi}{3}}$$

17 $\int_1^2 \frac{\ln(x)}{x^2}\, dx$ LET $u = \ln(x)$ $v = -\frac{1}{x}$

$du = \frac{1}{x}\, dx$ $dv = \frac{1}{x^2}\, dx$

$$= -\frac{\ln(x)}{x}\Big|_1^2 + \int_1^2 \frac{1}{x^2}\, dx$$

$\frac{1}{2}$

$$= -\frac{\ln(2)}{2} + \left[\frac{-1}{x}\right]_1^2 = \boxed{\frac{1}{2}\left(1 - \ln(2)\right)}$$

19 $\int_0^1 \frac{y}{e^{2y}}\, dy = \int_0^1 y e^{-2y}\, dy$ LET $u = y$ $v = -\frac{1}{2}e^{-2y}$

$du = dy$ $dv = e^{-2y}\, dy$

$$= -\frac{1}{2} y e^{-2y}\Big|_0^1 + \frac{1}{2}\int_0^1 e^{-2y}\, dy$$

$$= \frac{-1}{2e^2} + \frac{1}{2}\left[-\frac{1}{2}e^{-2y}\right]_0^1 = \frac{-1}{2e^2} - \frac{1}{4e^2} + \frac{1}{4} = \boxed{\frac{1}{4} - \frac{3}{4e^2}}$$

$\boxed{21}$ $\displaystyle\int_0^{1/2} \sin^{-1}(x)\,dx$

LET $u = \sin^{-1}(x)$ $\quad v = x$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx$ $\quad dv = dx$

$$= x\sin^{-1}(x)\Big|_0^{1/2} - \underbrace{\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx}$$

$\dfrac{1}{2}\cdot\dfrac{\pi}{6} - 0$

LET $u = 1 - x^2$

$du = -2x\,dx \Rightarrow -\frac{1}{2}\,du = x\,dx$

$x = 0 \Rightarrow u = 1$

$x = \frac{1}{2} \Rightarrow u = \frac{5}{4}$

$$-\frac{1}{2}\int_1^{3/4} \frac{1}{\sqrt{u}}\,du = -\sqrt{u}\,\Big|_1^{3/4} = 1 - \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{12} - \left(1 - \frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{12} - 1 + \frac{\sqrt{3}}{2}}$$

$\boxed{23}$ $\displaystyle\int_1^2 (\ln x)^2\,dx$

LET $u = (\ln x)^2$ $\quad v = x$

$du = \dfrac{2\ln(x)}{x}\,dx$ $\quad dv = dx$

(DONE IN CLASS)

$$= x(\ln x)^2\Big|_1^2 - 2\int_1^2 \ln(x)\,dx = 2(\ln 2)^2 - 2\Big[x\ln(x) - x\Big]_1^2$$

$$= 2\ln(2)^2 - 4\ln(2) + 4 - 2$$

$$= \boxed{2(\ln 2)^2 - 4\ln(2) + 2}$$

25 $\int \sin\sqrt{x}\,dx$ LET $w = \sqrt{x}$

i.e. $x = w^2$

$dx = 2w\,dw$

$\int 2w \sin(w)\,dw$ NOW LET $u = 2w$ $v = -\cos(w)$

$du = 2\,dw$ $dv = \sin(w)\,dw$

$= -2w\cos(w) + \int 2\cos(w)\,dw$

$= -2w\cos(w) + 2\sin(w) + C$

$= \boxed{-2\sqrt{x}\cos(\sqrt{x}) + 2\sin(\sqrt{x}) + C}$

27 $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\,d\theta$ LET $w = \theta^2$

$dw = 2\theta\,d\theta$

$\theta = \sqrt{\frac{\pi}{2}} \Rightarrow w = \frac{\pi}{2}$

$\theta = \sqrt{\pi} \Rightarrow w = \pi$

$\int_{\pi/2}^{\pi} \frac{1}{2} w \cos(w)\,dw$ NOW LET $u = \frac{1}{2}w$ $v = \sin(w)$

$du = \frac{1}{2}\,dw$ $dv = \cos(w)\,dw$

$= \frac{1}{2} w \sin(w) \Big|_{\pi/2}^{\pi} - \frac{1}{2}\int_{\pi/2}^{\pi} \sin(w)\,dw = -\frac{\pi}{4} - \frac{1}{2}\Big[-\cos(w)\Big]_{\pi/2}^{\pi}$

$= \boxed{-\frac{\pi}{4} - \frac{1}{2}}$

29 (a) REDUCTION FORMULA WITH $n = 2$:

$$\int \sin^2 x \, dx = \underbrace{-\frac{1}{2}\cos(x)\sin(x)}_{} + \frac{1}{2}\int dx$$

$$= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

(b) $$\int \sin^4(x)\,dx = -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \sin^2(x)\,dx$$

$$= -\frac{1}{4}\cos(x)\sin(x)\sin^2(x) + \frac{3}{4}\left[\frac{1}{2}x - \frac{1}{4}\sin(2x)\right]$$

$$= \boxed{-\frac{1}{8}\sin(2x)\sin^2(x) + \frac{3}{8}x - \frac{3}{16}\sin(2x) + C}$$

30 (a) $\int \cos^n(x)\,dx$ LET $u = \cos^{n-1}(x)$ $\quad v = \sin(x)$

$du = -(n-1)\cos^{n-2}(x)\sin(x)\,dx \quad dv = \cos(x)\,dx$

$$= \sin(x)\cos^{n-1}(x) + \underbrace{(n-1)\int \cos^{n-2}(x)\sin^2(x)\,dx}$$

$$+ (n-1)\int \cos^{n-2}(x)\left(1 - \cos^2(x)\right)dx$$

$$+ (n-1)\int \cos^{n-2}(x)\,dx - (n-1)\int \cos^n(x)\,dx$$

$$\therefore \int \cos^n(x)\,dx = \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)\,dx - (n-1)\int \cos^n(x)\,dx$$

$$\Rightarrow n\int \cos^n(x)\,dx = \sin(x)\cos^{n-1}(x) + (n-1)\int \cos^{n-2}(x)\,dx$$

$$\Rightarrow \int \cos^n(x)\,dx = \frac{1}{n}\sin(x)\cos^{n-1}(x) + \frac{(n-1)}{n}\int \cos^{n-2}(x)\,dx$$

(b) $\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}\int dx$

$$= \boxed{\frac{1}{4}\sin(2x) + \frac{1}{2}x + C}$$

(c) $\int \cos^4(x)\,dx = \frac{1}{4}\cos^3(x)\sin(x) + \frac{3}{4}\int \cos^2(x)\,dx$

$$= \boxed{\frac{1}{4}\cos^3(x)\sin(x) + \frac{3}{4}\left(\frac{1}{4}\sin(2x) + \frac{1}{2}x\right) + C}$$

33. $\int (\ln x)^n\,dx$ LET $u = (\ln x)^n$ $v = x$

$du = \frac{n(\ln x)^{n-1}}{x}$ $dv = dx$

$$= x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$$

✓

34. $\int x^n e^x\,dx$ LET $u = x^n$ $v = e^x$

$du = nx^{n-1}\,dx$ $dv = e^x\,dx$

$$= x^n e^x - n\int x^{n-1}e^x\,dx$$

✓

$\boxed{39}$ RECALL : $f_{AVE} = \dfrac{1}{b-a} \displaystyle\int_a^b f(x)\,dx$ [p. 232]

$$\therefore f_{AVE} = \frac{1}{3-1} \int_1^3 x^2 \ln(x)\,dx$$

LET $u = \ln(x)$ $\quad v = \frac{1}{3}x^3$

$du = \frac{1}{x}\,dx$ $\quad dv = x^2\,dx$

$$= \frac{1}{2}\left[\frac{1}{3}x^3 \ln(x) \Big|_1^3 - \frac{1}{3}\int_1^3 x^2\,dx \right]$$

$$= \frac{1}{2}\left[9\ln(3) - \frac{1}{3}\left[\frac{1}{3}x^3\right]_1^3 \right] =$$

$$= \frac{1}{2}\left[9\ln(3) - \frac{1}{3}\left(9 - \frac{1}{3}\right)\right] = \frac{1}{2}\left[9\ln(3) - \frac{26}{9}\right]$$

$$-3 + \frac{1}{9}$$

$$= \boxed{\frac{9}{2}\ln(3) - \frac{13}{9}}$$