JOHN ADAMSKI
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MATH 202

§5.8

3 $\lim_{x \to \frac{\pi}{2}^+} \frac{\cos(x)}{1-\sin(x)}$ $\frac{0}{0}$ IND. FORM!

$= \lim_{x \to \frac{\pi}{2}^+} \frac{-\sin(x)}{-\cos(x)} \begin{matrix} \longrightarrow -1 \\ \longrightarrow 0^+ \end{matrix} = \boxed{-\infty}$

ZERO FROM ABOVE

6 $\lim_{t \to 0} \frac{e^{3t}-1}{t}$ $\frac{0}{0}$ IND. FORM!

$= \lim_{t \to 0} \frac{3e^{3t}}{1} = \boxed{3}$

9 $\lim_{x \to 0^+} \frac{\ln(x)}{x}$ $\frac{0}{0}$ IND FORM!

$= \lim_{x \to 0^+} \frac{1/x}{1} = \boxed{\infty}$

12 $\lim_{x \to 1} \frac{\ln(x)}{\sin(\pi x)}$ $\frac{0}{0}$ IND FORM!

$= \lim_{x \to 1} \frac{1/x}{\pi \cos(\pi x)} = \boxed{-\frac{1}{\pi}}$

$\boxed{15}$ $\lim_{x \to \infty} \frac{x}{\ln(1+2e^x)}$ $\frac{\infty}{\infty}$ IND FORM!

$$= \lim_{x \to \infty} \frac{1}{\frac{1}{1+2e^x}(2e^x)} = \lim_{x \to \infty} \frac{1+2e^x}{2e^x}$$

MAYBE YOU SEE THE LIMIT ALREADY....

IF NOT, APPLY L'HOSPITAL AGAIN!

$$= \lim_{x \to \infty} \frac{2e^x}{2e^x} = \boxed{1}$$

$\boxed{18}$ $\lim_{x \to 0} \frac{x}{\tan^{-1}(4x)}$ $\frac{0}{0}$ IND FORM!

$$= \lim_{x \to 0} \frac{1}{\frac{4}{1+(4x)^2}} = \lim_{x \to 0} \frac{1+16x^2}{4} = \boxed{\frac{1}{4}}$$

$\boxed{21}$ $\lim_{x \to 0^+} \sqrt{x}\,\ln(x) = \lim_{x \to 0^+} \frac{\ln(x)}{x^{-1/2}}$ $-\frac{\infty}{\infty}$ IND FORM!

$$= \lim_{x \to 0^+} \frac{1/x}{-\frac{1}{2}x^{-3/2}} = \lim_{x \to 0^+} -2\sqrt{x} = \boxed{0}$$

$\boxed{24}$ $\lim_{x\to 0^+} \sin(x)\ln(x) = \lim_{x\to 0^+} \dfrac{\ln(x)}{\frac{1}{\sin(x)}}$ $\quad -\dfrac{\infty}{\infty}$ IND FORM!

$$= \lim_{x\to 0^+} \frac{1/x}{-\frac{\cos(x)}{\sin^2(x)}} = \lim_{x\to 0^+} -\frac{\sin^2(x)}{x\cos(x)} \quad \frac{0}{0} \text{ IND FORM!}$$

$$= \lim_{x\to 0^+} -\frac{2\sin(x)\cos(x)}{\cos(x) - x\sin(x)} = \boxed{0}$$

$\boxed{27}$ $\lim_{x\to\infty} (xe^{1/x} - x) = \lim_{x\to\infty} x(e^{1/x} - 1)$

$$= \lim_{x\to\infty} \frac{e^{1/x} - 1}{1/x} \quad \frac{0}{0} \text{ IND FORM!}$$

$$= \lim_{x\to\infty} \frac{e^{1/x}(-1/x^2)}{(-1/x^2)} = \boxed{1}$$

$\boxed{30}$ $\lim_{x\to 1} \left(\dfrac{1}{\ln(x)} - \dfrac{1}{x-1}\right) = \lim_{x\to 1} \dfrac{x-1-\ln(x)}{\ln(x)(x-1)}$ $\quad \dfrac{0}{0}$!

$$= \lim_{x\to 1} \frac{1 - 1/x}{\frac{1}{x}(x-1) + \ln(x)} = \lim_{x\to 1} \frac{1 - 1/x}{1 - 1/x + \ln(x)} \quad \frac{0}{0} \text{ !}$$

$$= \lim_{x\to 1} \frac{1/x^2}{1/x^2 + 1/x} = \boxed{\frac{1}{2}}$$

$\boxed{33}$ $\lim_{x \to 0} (1-2x)^{1/x} = L$

$$\Rightarrow \ln(L) = \lim_{x \to 0} \frac{\ln(1-2x)}{x} \qquad \frac{0}{0} \text{ IND FORM!}$$

$$= \lim_{x \to 0} \frac{\frac{-2}{1-2x}}{1} = -2$$

$$\therefore L = e^{\ln(L)} = \boxed{e^{-2}}$$

$\boxed{36}$ $\lim_{x \to \infty} x^{\ln(2)/(1+\ln(x))} = L$

$$\ln(L) = \lim_{x \to \infty} \frac{\ln(2)\ln(x)}{1 + \ln(x)} \qquad \frac{\infty}{\infty} \text{ IND FORM!}$$

$$= \lim_{x \to \infty} \frac{\frac{\ln(2)}{x}}{\frac{1}{x}} = \lim_{x \to \infty} \ln(2) = \ln(2)$$

$$\therefore L = e^{\ln(L)} = e^{\ln(2)} = \boxed{2}$$

$\boxed{39}$ $\lim_{x \to \infty} \dfrac{e^x}{x^n}$ $\dfrac{\infty}{\infty}$ IND FORM!

$$= \lim_{x \to \infty} \frac{e^x}{n\,x^{n-1}} = \lim_{x \to \infty} \frac{e^x}{n(n-1)\,x^{n-2}} = \cdots$$

$$\cdots = \lim_{x \to \infty} \frac{e^x}{n(n-1)\cdots(2)(1)\,x^0} = \infty$$

$\boxed{40}$ $\lim_{x \to \infty} \dfrac{\ln(x)}{x^p}$ $\dfrac{\infty}{\infty}$ IND FORM

$$= \lim_{x \to \infty} \frac{1/x}{p\,x^{p-1}} = \lim_{x \to \infty} \frac{1}{p\,x^p} = 0$$

$\boxed{41}$ $\lim_{n \to \infty} \left(1 + \dfrac{r}{n}\right)^n = L$

$$\ln(L) = \lim_{n \to \infty} n \ln\left(1 + \frac{r}{n}\right) = \lim_{n \to \infty} \frac{\ln\left(1 + \frac{r}{n}\right)}{\frac{1}{n}} \quad \frac{0}{0}!$$

$$= \lim_{n \to \infty} \frac{\frac{1}{1 + \frac{r}{n}}\left(-r/n^2\right)}{-1/n^2} = \lim_{n \to \infty} \frac{r}{1 + \frac{r}{n}} = r$$

$$\therefore\ L = e^{\ln(L)} = e^r$$

$$\therefore\ \lim_{n \to \infty} A_0\left(1 + \frac{r}{n}\right)^{nt} = A_0 e^{rt}$$

47 $\lim_{x \to 0} \dfrac{f(2+3x) + f(2+5x)}{x}$ $\quad \dfrac{0}{0}$ IND FORM!

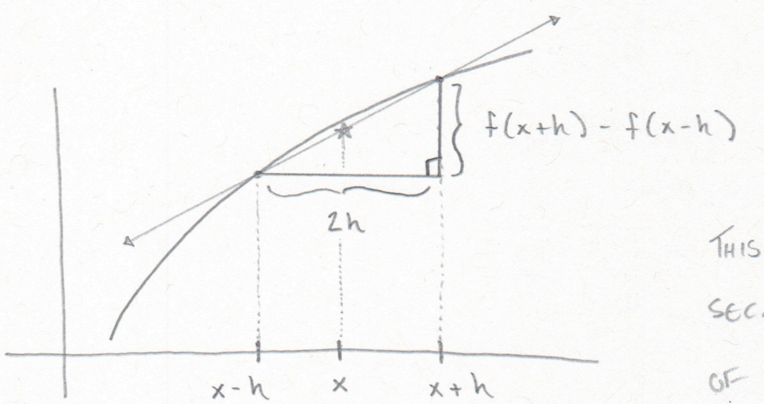
$$= \lim_{x \to 0} \frac{3f'(2+3x) + 5f'(2+5x)}{1}$$

$$= 3f'(2) + 5f'(2) = 21 + 35 = \boxed{56}$$

49 $\lim_{h \to 0} \dfrac{f(x+h) - f(x-h)}{2h}$ $\quad \dfrac{0}{0}$ IND. FORM!

$$= \lim_{h \to 0} \frac{\frac{d}{dh}\left[f(x+h) - f(x-h)\right]}{\frac{d}{dh}\left[2h\right]} = \lim_{h \to 0} \frac{f'(x+h) + f'(x+h)}{2}$$

$$= \lim_{h \to 0} f'(x+h) = f'(x)$$ ✓



THIS IS STILL A LIMIT OF SLOPES OF SECANTS THAT APPROACH THE SLOPE OF THE TANGENT LINE AT X.

(i.e. DERIVATIVE)