MATH 202

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§ 5.7

1. $\sinh(0) = \dfrac{e^0 - e^0}{2} = \boxed{0}$

$\cosh(0) = \dfrac{e^0 + e^0}{2} = \boxed{1}$

3. (a) $\sinh(\ln(2)) = \dfrac{e^{\ln(2)} - e^{-\ln(2)}}{2} = \dfrac{2 - \frac{1}{2}}{2} = \boxed{\dfrac{3}{4}}$

(b) $\sinh(2) = \dfrac{e^2 - e^{-2}}{2} = \dfrac{1}{2}\left(\dfrac{e^4}{e^2} - \dfrac{1}{e^2}\right) = \boxed{\dfrac{e^4 - 1}{2e^2}}$

5. (a) $\operatorname{sech}(0) = \dfrac{2}{e^0 + e^0} = \boxed{1}$

(b) $\cosh^{-1}(1) = \boxed{0}$ BECAUSE $\cosh(0) = 1$ & $0 \in [0, \infty)$

7. $\sinh(-x) = \dfrac{e^{(-x)} - e^{-(-x)}}{2} = \dfrac{e^{-x} - e^{x}}{2} = \dfrac{-(e^{x} - e^{-x})}{2}$

$= -\sinh(x)$ ✓

8 $\cosh(-x) = \dfrac{e^{(-x)} + e^{-(-x)}}{2} = \dfrac{e^{x} + e^{-x}}{2} = \cosh(x)$ ✓

9 $\cosh(x) + \sinh(x) = \dfrac{e^{x} + e^{-x}}{2} + \dfrac{e^{x} - e^{-x}}{2}$

$= \dfrac{2e^{x}}{2} = e^{x}$ ✓

10 $\cosh(x) - \sinh(x) = \dfrac{e^{x} + e^{-x}}{2} - \dfrac{e^{x} - e^{-x}}{2}$

$= \dfrac{2e^{-x}}{2} = e^{-x}$ ✓

15 $(\cosh(x) + \sinh(x))^{n} = (e^{x})^{n}$ BY EXERCISE # 9

AND $\cosh(nx) + \sinh(nx) = \dfrac{e^{nx} + e^{-nx}}{2} + \dfrac{e^{nx} - e^{-nx}}{2}$

$= \dfrac{2e^{nx}}{2} = e^{nx}$ SAME! ✓

$\boxed{16}$ $\sinh(x) = \frac{3}{4}$

IDENTITY: $\cosh^2(x) - \sinh^2(x) = 1$

$$\Rightarrow \cosh(x) = \sqrt{1 + \sinh^2(x)}$$

$$= \sqrt{1 + \left(\frac{3}{4}\right)^2} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$\therefore$

$$\boxed{\begin{array}{ll} \sinh(x) = \frac{3}{4} & \operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{4}{3} \\ \cosh(x) = \frac{5}{4} & \operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{4}{5} \\ \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{3}{5} & \coth(x) = \frac{1}{\tanh(x)} = \frac{5}{3} \end{array}}$$

$\boxed{17}$ $\tanh(x) = \frac{4}{5}$

IDENTITY: $1 - \tanh^2(x) = \operatorname{sech}^2(x)$

$$\Rightarrow \operatorname{sech}(x) = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\therefore \cosh(x) = \frac{5}{3}$$

IDENTITY: $\cosh^2(x) - \sinh^2(x) = 1$

$$\Rightarrow \sinh(x) = \sqrt{\cosh^2(x) - 1} = \sqrt{\left(\frac{5}{3}\right)^2 - 1} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

$\therefore$

$$\boxed{\begin{array}{ll} \sinh(x) = \frac{4}{3} & \operatorname{csch}(x) = \frac{3}{4} \\ \cosh(x) = \frac{5}{3} & \operatorname{sech}(x) = \frac{3}{5} \\ \tanh(x) = \frac{4}{5} & \coth(x) = \frac{5}{4} \end{array}}$$

27 $f(x) = x\cosh(x)$

$$\boxed{f'(x) = \cosh(x) + x\sinh(x)}$$ (PRODUCT RULE)

29 $h(x) = \sinh(x^2)$

$$\boxed{h'(x) = \cosh(x^2)\cdot 2x}$$ (CHAIN RULE)

31 $h(t) = \coth\sqrt{1+t^2} = \left[\tanh\left(\sqrt{1+t^2}\right)\right]^{-1}$

$$h'(t) = -\left[\tanh\left(\sqrt{1+t^2}\right)\right]^{-2}\cdot \operatorname{sech}^2\left(\sqrt{1+t^2}\right)\cdot \frac{2t}{2\sqrt{1+t^2}}$$

$$= \frac{-\cosh^2\left(\sqrt{1+t^2}\right)}{\sinh^2\left(\sqrt{1+t^2}\right)}\cdot\frac{1}{\cosh^2\left(\sqrt{1+t^2}\right)}\cdot\frac{t}{\sqrt{1+t^2}}$$

$$= \boxed{\frac{t}{\sqrt{1+t^2}\,\sinh^2\left(\sqrt{1+t^2}\right)}} \text{ or } \boxed{\frac{t\operatorname{csch}^2\left(\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}}$$

33 $H(t) = \tanh(e^t)$

$$\boxed{H'(t) = \operatorname{sech}^2(e^t)\cdot e^t}$$

35 $y = e^{\cosh(3x)}$

$$y' = e^{\cosh(3x)}\cdot\sinh(3x)\cdot 3$$

$$= \boxed{3e^{\cosh(3x)}\sinh(3x)}$$

$\boxed{37}$ $y = \tanh^{-1}\sqrt{x} \iff \tanh(y) = \sqrt{x}$

$$\Rightarrow \operatorname{sech}^2(y)\, y' = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow y' = \frac{1}{2\sqrt{x}\operatorname{sech}^2(y)} = \frac{1}{2\sqrt{x}\,(1 - \underbrace{\tanh^2(y)}_{=(\sqrt{x})^2 = x})}$$

$$\boxed{y' = \frac{1}{2\sqrt{x}\,(1-x)}}$$

$\boxed{39}$ $y = x \sinh^{-1}\left(\frac{x}{3}\right) - \sqrt{9+x^2}$

$$y' = \sinh^{-1}\left(\frac{x}{3}\right) + x\frac{1}{\sqrt{1+\left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} - \frac{2x}{2\sqrt{9+x^2}}$$

$$\frac{x}{3}\frac{1}{\sqrt{\frac{9+x^2}{9}}} = \frac{x}{3}\sqrt{\frac{9}{9+x^2}} = \frac{x}{3}\cdot\frac{3}{\sqrt{9+x^2}} = \frac{x}{\sqrt{9+x^2}}$$

$$\therefore y' = \sinh^{-1}\left(\frac{x}{3}\right) + \frac{x}{\sqrt{9+x^2}} - \frac{x}{\sqrt{9+x^2}}$$

$$\boxed{y' = \sinh^{-1}\left(\frac{x}{3}\right)}$$

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$$y = \coth^{-1}\sqrt{x^2+1}$$

$$y' = \frac{1}{1-\left(\sqrt{x^2+1}\right)^2} \cdot \frac{2x}{2\sqrt{x^2+1}}$$

$$y' = \frac{1}{x^2} \cdot \frac{x}{\sqrt{x^2+1}} = \boxed{\frac{1}{x\sqrt{x^2+1}}}$$

43 (a) CURVE MEETS RIGHT POLE WHEN $x = 7$

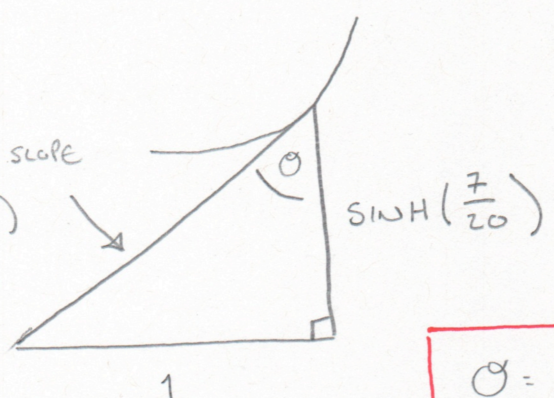
$$f(x) = 20\cosh\left(\frac{x}{20}\right)$$

$$f'(x) = \frac{20\sinh\left(\frac{x}{20}\right)}{20} = \sinh\left(\frac{x}{20}\right)$$

$$\therefore \boxed{f'(7) = \sinh\left(\frac{7}{20}\right)}$$

(b)

THIS LINE HAS SLOPE $\sinh\left(\frac{7}{20}\right)$



$$\boxed{\theta = \arctan\left(\operatorname{csch}\left(\frac{7}{20}\right)\right)}$$

$\boxed{46}$ $\lim_{x \to \infty} \frac{\sinh(x)}{e^x} = \lim_{x \to \infty} \frac{e^x - e^{-x}}{2} \cdot \frac{1}{e^x}$

$= \lim_{x \to \infty} \frac{1 - e^{-2x}}{2} = \boxed{\frac{1}{2}}$

$\boxed{47}$ $y = \cosh(x)$

$y' = \sinh(x) = 1$ (SOLVE FOR x)

$\frac{e^x - e^{-x}}{2} = 1 \Rightarrow e^x - e^{-x} = 2$

THIS IS A QUADRATIC IN e^x

(LET'S SEE HOW!)

$e^x - e^{-x} = 2 \Rightarrow \left[e^x - 2 - e^{-x} = 0 \right] e^x$ (TO GET RID OF NEGATIVE EXP.)

$\Rightarrow (e^x)^2 - 2(e^x) - 1 = 0$

(OR $w^2 - 2w - 1 = 0$ WHERE $w = e^x$)

$\underbrace{0 < e^x}_{\text{(MUST BE POSITIVE)}} = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

(MUST BE POSITIVE) $\longrightarrow$ REJECT "−"

$\therefore e^x = 1 + \sqrt{2}$

$\Rightarrow x = \ln(1 + \sqrt{2})$

$$y = \cosh\left(\ln(1+\sqrt{2})\right)$$

$$= \frac{e^{\ln(1+\sqrt{2})} + e^{-\ln(1+\sqrt{2})}}{2}$$

$$= \left(1+\sqrt{2} + \frac{1}{1+\sqrt{2}}\right)\cdot\frac{1}{2}$$

$$= \frac{(1+\sqrt{2})^2+1}{2(1+\sqrt{2})} = \frac{(1+2\sqrt{2}+2)+1}{2(1+\sqrt{2})} = \frac{4+2\sqrt{2}}{2(1+\sqrt{2})}$$

$$= \frac{2(2+\sqrt{2})}{2(1+\sqrt{2})} = \frac{2+\sqrt{2}}{1+\sqrt{2}}\left(\frac{1-\sqrt{2}}{1-\sqrt{2}}\right) = \frac{2-2\sqrt{2}+\sqrt{2}-2}{1-\sqrt{2}+\sqrt{2}-2}$$

$$= \frac{-\sqrt{2}}{-1} = \sqrt{2}$$

$$\therefore \boxed{(x,y) = \left(\ln(1+\sqrt{2}), \sqrt{2}\right)}$$