§ 5.6

1 (a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$

BECAUSE $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ AND $-\frac{\pi}{2} \le \frac{\sqrt{3}}{2} \le \frac{\pi}{2}$

(b) $\cos^{-1}(-1) = \boxed{\pi}$

BECAUSE $\cos(\pi) = -1$ AND $0 \le \pi \le \pi$

3 (a) $\tan^{-1}(\sqrt{3}) = \boxed{\frac{\pi}{3}}$

BECAUSE $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ AND $0 \le \frac{\pi}{3} \le \pi$

(b) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-\frac{\pi}{4}}$

BECAUSE $\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ OR $-\frac{\sqrt{2}}{2}$

AND $-\frac{\pi}{2} \le -\frac{\pi}{4} \le \frac{\pi}{2}$

5 (a) $\sin(\sin^{-1}(0.7)) = \boxed{0.7}$

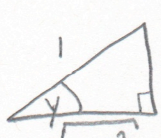
(b) $\tan^{-1}\left(\tan\left(\frac{4\pi}{3}\right)\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right) = \boxed{\frac{\pi}{3}}$

TAN IS PERIODIC W/ PERIOD π

$-\frac{\pi}{2} \le \frac{\pi}{3} \le \frac{\pi}{2}$

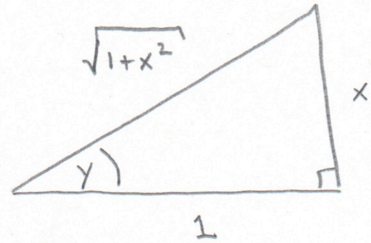
8 $\tan(\sin^{-1}(x))$

LET $y = \sin^{-1}(x)$ i.e. $\sin(y) = x$



$\tan(\sin^{-1}(x)) = \tan(y) = \boxed{\frac{x}{\sqrt{1-x^2}}}$

[9] $\sin(\tan^{-1}(x))$



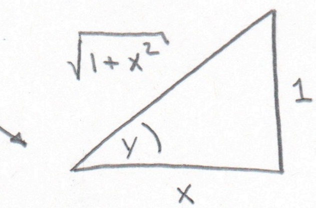
LET $y = \tan^{-1}(x)$

SO $\tan(y) = x$

THEN $\sin(\tan^{-1}(x)) = \sin(y) = \boxed{\dfrac{x}{\sqrt{1+x^2}}}$

[13] LET $y = \cot^{-1}(x)$ SO $\underbrace{\cot(y) = x}$ i.e. $\dfrac{1}{\tan(y)} = x$

THEN $\dfrac{-1}{\tan^2(y)} \cdot \sec^2(y) \cdot y' = 1$

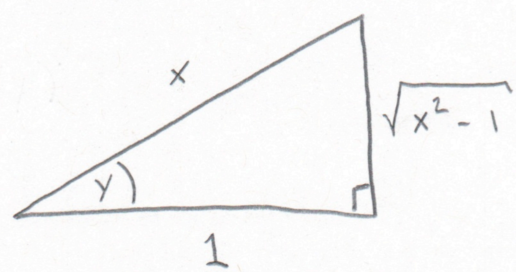


$\Rightarrow y' = \dfrac{-\tan^2(y)}{\sec^2(y)} = -\sin^2(y) = \dfrac{-1}{1+x^2}$ ✓

[14] LET $y = \sec^{-1}(x)$ SO $\sec(y) = x$ i.e. $\dfrac{1}{\cos(y)} = x$

THEN $\dfrac{d}{dx}\left[\dfrac{1}{\cos(y)} = x\right] \Rightarrow \dfrac{-1}{\cos^2(y)} \sin(y)\, y' = 1$

$\Rightarrow y' = \dfrac{\cos^2(y)}{\sin(y)}$



$y' = \left(\dfrac{1}{x}\right)^2 \dfrac{x}{\sqrt{x^2-1}} = \dfrac{1}{x\sqrt{x^2-1}}$ ✓

16 $y = \sqrt{\tan^{-1}(x)}$

$$y' = \frac{1}{2\sqrt{\tan^{-1}(x)}} \cdot \frac{d}{dx} \tan^{-1}(x)$$

$$= \boxed{\frac{1}{2\sqrt{\tan^{-1}(x)}} \cdot \frac{1}{1+x^2}}$$

17 $y = \tan^{-1}\sqrt{x}$

$$y' = \frac{1}{1+\sqrt{x}^2} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

19 $y = \sin^{-1}(2x+1)$

$$y' = \frac{1}{\sqrt{1-(2x+1)^2}} \cdot 2 = \frac{2}{\sqrt{1-(4x^2+4x+1)}} = \frac{2}{\sqrt{4(-x^2-x)}}$$

$$= \boxed{\frac{1}{\sqrt{-x^2-x}}}$$

23 $y = \cos^{-1}(e^{2x})$

$$y' = \frac{-1}{\sqrt{1-(e^{2x})^2}} \cdot 2e^{2x} = \boxed{\frac{-2e^{2x}}{\sqrt{1-e^{4x}}}}$$

25 $y = \arctan(\cos\theta)$

$$y' = \frac{1}{1+\cos^2\theta} \cdot -\sin\theta = \boxed{\frac{-\sin\theta}{1+\cos^2\theta}}$$

28 $y = \tan^{-1}\left(\frac{x}{a}\right) + \ln\sqrt{\frac{x-a}{x+a}}$

$$y = \tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}\left[\ln(x-a) - \ln(x+a)\right]$$

$$y' = \underbrace{\frac{1}{1+\left(\frac{x}{a}\right)^2} \cdot \frac{1}{a}}_{\frac{1}{a+\frac{x^2}{a}}} + \frac{1}{2}\underbrace{\left[\frac{1}{x-a} - \frac{1}{x+a}\right]}_{\frac{2a}{x^2-a^2}}$$

$$y' = \frac{a}{a^2+x^2} + \frac{a}{x^2-a^2} = a\left(\frac{1}{a^2+x^2} + \frac{1}{x^2-a^2}\right)$$

$$\boxed{y' = \frac{2ax^2}{x^4-a^4}}$$

$\boxed{30}$ $f(x) = \text{ARCSIN}(e^x)$

$$\uparrow$$

$$\underbrace{-1 \le e^x}_{} \le 1 \quad \Rightarrow \quad x \le \ln(1) = 0$$

NO NEED TO WORRY ABOUT THIS INEQUALITY

$\therefore$ DOMAIN : $(-\infty, 0]$

RANGE : $(0, \frac{\pi}{2}]$

$$f'(x) = \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = \frac{e^x}{\sqrt{1-e^{2x}}}$$

DOMAIN : $(-\infty, 0)$

RANGE : $(0, \infty)$

(NOTICE THE SLIGHT DIFFERENCE)

$\boxed{31}$ $g(x) = \cos^{-1}(3-2x)$

$$\uparrow$$

$$-1 \le 3-2x \le 1$$
$$-4 \le -2x \le -2$$
$$2 \ge x \ge 1$$

DOMAIN : $[1, 2]$

RANGE : $[0, \pi]$

$$g'(x) = \frac{-1}{\sqrt{1-(3-2x)^2}} \cdot (-2) = \frac{2}{\sqrt{1-(3-2x)^2}}$$

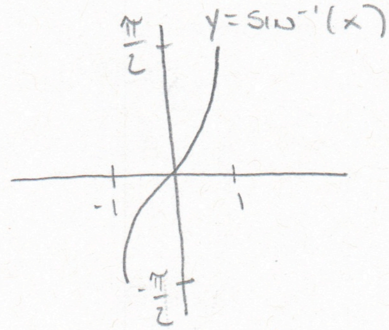
(SLIGHTLY DIFFERENT)

$$0 < 1-(3-2x)^2$$
$$(3-2x)^2 < 1$$
$$|3-2x| < 1 \quad \Rightarrow \quad -1 < 3-2x < 1$$
$$2 > x > 1$$

DOMAIN $(1, 2)$

RANGE $[2, \infty)$

35 $\lim_{x \to -1^+} \sin^{-1}(x)$



$\longrightarrow \boxed{-\frac{\pi}{2}}$

36 $\lim_{x \to \infty} \arccos\left(\frac{1+x^2}{1+2x^2}\right) = \arccos\left(\lim_{x \to \infty} \frac{1+x^2}{1+2x^2}\right)$

(PROVIDED THE LIMIT EXISTS)

$= \arccos\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$ BECAUSE $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

AND $0 \le \frac{\pi}{3} \le \pi$

37 $\lim_{x \to \infty} \arctan(e^x) = \arctan\left(\lim_{x \to \infty} e^x\right) = \boxed{\frac{\pi}{2}}$

BECAUSE $\lim_{w \to \infty} \arctan(w) = \frac{\pi}{2}$

AND IF $w = e^x$ THEN $w \to \infty$ AS $x \to \infty$.

38 $\lim_{x \to 0^+} \tan^{-1}(\ln(x)) = \tan^{-1}\left(\lim_{x \to 0^+} \ln(x)\right) = \boxed{-\frac{\pi}{2}}$

BECAUSE $\lim_{w \to -\infty} \text{ACTAN}(w) = -\frac{\pi}{2}$

AND IF $w = \ln(x)$ THEN $w \to -\infty$ AS $x \to 0^+$