JOHN ADAMSKI
6/9/2012

MATH 202

§5.5

$\boxed{1}$ $P(t) = P(0)e^{kt}$

$= (2)e^{(0.7944)t}$

$\therefore P(6) = 2e^{0.7944 \cdot 6}$

$= 234.99099\ldots$

$\approx \boxed{235}$

$\boxed{3}$ $P(t) = P(0)e^{kt}$, $P(0) = 100$, $P(1) = 420$

$\therefore P(1) = 100e^{k(1)} = 420$

$e^k = \frac{42}{10}$

$k = \ln(42) - \ln(10)$ or $\ln\left(\frac{42}{10}\right)$

So $P(t) = 100e^{\ln\left(\frac{42}{10}\right)t}$

(a) $\boxed{P(t) = 100\left(\frac{42}{10}\right)^t}$

(b) $P(3) = \boxed{100\left(\frac{42}{10}\right)^3} = 7408.8 \approx \boxed{7409}$

(c) $P'(t) = 100\left(\frac{42}{10}\right)^t \ln\left(\frac{42}{10}\right)$

$P'(3) = 7408.8 \ln\left(\frac{42}{10}\right) \approx \boxed{10632 \text{ BACTERIA/HR}}$

(d) $P(t) = 100\left(\frac{42}{10}\right)^t = 10{,}000$

$\left(\frac{42}{10}\right)^t = 100 \Rightarrow t = \dfrac{\ln(100)}{\ln\left(\frac{42}{10}\right)}$ or $\boxed{\dfrac{\ln(100)}{\ln(42) - \ln(10)}}$

4 $P(t) = P(0)e^{kt}$

$P(2) = P(0)e^{2k} = 600$ (EQ 1)

$P(8) = P(0)e^{8k} = 75000$ (EQ 2)

$(\text{EQ } 2) \div (\text{EQ } 1) \longrightarrow e^{6k} = 125 \longrightarrow 6k = \ln(125)$

$\Rightarrow k = \dfrac{\ln(125)}{6}$

$\therefore P(2) = P(0)e^{\frac{\ln(125)}{6}\cdot 2} = P(0)\,125^{1/3} = P(0)\cdot 5 = 600$

$\Rightarrow \boxed{P(0) = 120}$ (a)

$\boxed{P(t) = 120\,(125)^{t/6}}$ (b)

9 $\boxed{m(t) = 100\left(\tfrac{1}{2}\right)^{t/30}}$ (a)

$m(100) = 100\left(\tfrac{1}{2}\right)^{100/30} = \dfrac{100}{(\sqrt[3]{2})^{10}} \approx \boxed{9.92\ g}$ (b)

$m(t) = 100\left(\tfrac{1}{2}\right)^{t/30} = 1$

$\left(\tfrac{1}{2}\right)^{t/30} = \dfrac{1}{100}$

$\dfrac{t}{30}\ln\left(\tfrac{1}{2}\right) = \ln\left(\tfrac{1}{100}\right)$

$t = \dfrac{30\ln\left(\frac{1}{100}\right)}{\ln\left(\frac{1}{2}\right)} = \boxed{\dfrac{30\ln(100)}{\ln(2)}\ \text{YEARS}}$ (c)

≈ 199.32 YEARS

10 $m(t) = m_0(.945)^t$

$$\frac{m_0}{2} = m_0(.945)^t \longrightarrow \frac{1}{2} = (.945)^t$$

$$\ln\left(\tfrac{1}{2}\right) = t \ln(.945)$$

$$t = \boxed{\frac{\ln(\frac{1}{2})}{\ln(.945)} \approx 12.253 \text{ YEARS}} \quad \text{(a)}$$

20% OF m_0

$$m(t) = m_0(.945)^t = \frac{m_0}{5} \longrightarrow .945^t = \frac{1}{5}$$

$$t = \boxed{\frac{\ln(\frac{1}{5})}{\ln(.945)} \approx 28.45 \text{ YEARS}} \quad \text{(b)}$$

12 $\boxed{y = e^{2x}}$ (THEN $y' = 2e^{2x} = 2y$ ✓)

13 $T' = k(T - T_s)$; LET $y = T - T_s$ THEN $y' = ky$

$\Rightarrow y(t) = y(0)e^{kt}$; $y(0) = T(0) - T_s = 185 - 75 = 110$

$T(30) = 150 \Rightarrow y(30) = 150 - 75 = 75$

$$y(30) = 110e^{30k} = 75$$

$$e^{30k} = \frac{75}{110} = \frac{15}{22}$$

$$30k = \ln\left(\tfrac{15}{22}\right)$$

$$k = \frac{\ln(15) - \ln(22)}{30}$$

$$\therefore y(t) = 110e^{\ln(\frac{15}{22})t/30}$$

$$= 110\left(\tfrac{15}{22}\right)^{t/30}$$

T_s

$$T(t) = 110\left(\tfrac{15}{22}\right)^{t/30} + 75$$

$$\Rightarrow \boxed{T(45) = 110\left(\tfrac{15}{22}\right)^{45/30} + 75 \approx 136.93\,^\circ F} \quad \text{(a)}$$

$$T(t) = 110\left(\frac{15}{22}\right)^{t/30} + 75 = 100$$

$$\left(\frac{15}{22}\right)^{t/30} = \frac{25}{110}$$

(b)

$$\frac{t}{30}\ln\left(\frac{15}{22}\right) = \ln\left(\frac{25}{110}\right) \longrightarrow \boxed{t = \frac{30\left(\ln(25) - \ln(110)\right)}{\ln(15) - \ln(22)} \approx 116.05 \text{ MIN}}$$

15

$$T(t) = (-15)e^{kt} + 20$$

$$T(25) = (-15)e^{25K} + 20 = 10$$

$$e^{25k} = \frac{-10}{-15} = \frac{2}{3}$$

$$K = \frac{\ln(2) - \ln(3)}{25} \approx -0.0162$$

$$\therefore T(t) = (-15)\left(\frac{2}{3}\right)^{t/25} + 20$$

(a)

$$T(50) = (-15)\left(\frac{2}{3}\right)^{50/25} + 20 = 20 - \frac{4 \cdot 15}{9} = \boxed{13\tfrac{1}{3}\ {}^\circ C}$$

$$T(t) = (-15)\left(\frac{2}{3}\right)^{t/25} + 20 = 15$$

$$\left(\frac{2}{3}\right)^{t/25} = \frac{1}{3}$$

$$\frac{t}{25}\ln\left(\frac{2}{3}\right) = \ln\left(\frac{1}{3}\right)$$

(b)

$$\boxed{t = \frac{25\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{2}{3}\right)} \approx 67.74 \text{ MIN}}$$

$\boxed{16}$ $T(t) = (T_0 - T_s)e^{kt} + T_s \Rightarrow T(t) = (75)e^{kt} + 20$

$T_0 = 95$, $T_s = 20$

$\therefore\ T'(t) = 75ke^{kt} = k\left[75e^{kt}\right] = -1$

NOTE: WHEN $T = 70$, THIS EQUALS 50

$\Rightarrow\ 50k = -1 \Rightarrow k = -\frac{1}{50} = -.02$

$\therefore\ T(t) = (75)e^{-.02t} + 20 = 70$

$$e^{-.02t} = \frac{50}{75} = \frac{2}{3}$$

$$-.02t = \ln\left(\tfrac{2}{3}\right)$$

$$\boxed{t = \frac{\ln\left(\frac{2}{3}\right)}{-.02} = 50\left(\ln(3) - \ln(2)\right) \approx 20.27 \text{ MIN.}}$$