MATH 202 HW #3

§5.4

[3] $5^{\sqrt{7}} = \left(e^{\ln(5)}\right)^{\sqrt{7}} = \boxed{e^{\sqrt{7}\ln(5)}}$

[4] $10^{x^2} = \left(e^{\ln(10)}\right)^{x^2} = \boxed{e^{x^2\ln(10)}}$

[5] $\cos(x)^{x} = \left(e^{\ln(\cos(x))}\right)^{x} = \boxed{e^{x\ln(\cos(x))}}$

[6] $x^{\cos(x)} = \left(e^{\ln(x)}\right)^{\cos(x)} = \boxed{e^{\cos(x)\ln(x)}}$

[7] (a) $\log_{10} 1000 = \boxed{3} \quad \because \quad 10^3 = 1000$

(b) $\log_2 \frac{1}{16} = \boxed{-4} \quad \because \quad 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

[10] (a) $\log_a \frac{1}{a} = \boxed{-1} \quad \because \quad a^{-1} = \frac{1}{a}$

(b) $10^{\log_{10} 4 + \log_{10} 7} = 10^{\log_{10} 4} \cdot 10^{\log_{10} 7}$

$= 4 \cdot 7 = \boxed{28}$

17 $f(1) = 6 \longrightarrow Ca^1 = 6 \quad (1)$

$f(3) = 24 \longrightarrow Ca^3 = 24 \quad (2)$

$(2) \div (1) \longrightarrow a^2 = 4 \Rightarrow a = \pm 2$

REJECT NEG.

$\therefore f(x) = C \cdot 2^x$. THEN $f(1) = C \cdot 2 = 6 \Rightarrow C = 3$

$\therefore \boxed{f(x) = 3 \cdot 2^x}$

18 $f(0) = 2 \longrightarrow Ca^0 = 2 \Rightarrow C = 2$

$f(2) = \frac{2}{9} \longrightarrow 2a^2 = \frac{2}{9} \Rightarrow a^2 = \frac{1}{9} \Rightarrow a = \pm \frac{1}{3}$

REJECT NEG.

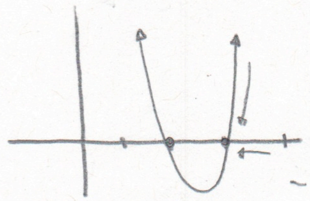
$\therefore \boxed{f(x) = 2\left(\frac{1}{3}\right)^x}$

21 $\lim_{t \to \infty} 2^{-t^2} \xrightarrow[\text{SINCE } -t^2 \to -\infty \text{ AS } t \to \infty]{} = \lim_{s \to -\infty} 2^s = \boxed{0}$

22 $\lim_{x \to 3^+} \log_{10}(x^2 - 5x + 6)$

NOTE: $x^2 - 5x + 6 = (x-3)(x-2) \longrightarrow 0^+$

AS $x \longrightarrow 3^+$



$= \lim_{t \to 0^+} \log_{10}(t) = \boxed{-\infty}$

23 $h(t) = t^3 - 3t$

$\boxed{h'(t) = 3t^2 - 3}$

25 $y = 5^{-1/x} = e^{-\frac{1}{x}\ln(5)}$

$y' = e^{-\frac{1}{x}\ln(5)}\left(\frac{\ln(5)}{x^2}\right)$

$$\boxed{y' = \frac{5^{-1/x}\ln(5)}{x^2}}$$

27 $f(u) = (2^u + 2^{-u})^{10}$

$\boxed{f'(u) = 10(2^u + 2^{-u})^9\left(2^u\ln(2) - 2^{-u}\ln(2)\right)}$

29 $f(x) = \log_3(x^2 - 4)$

$$\boxed{f'(x) = \frac{2x}{(x^2-4)\ln(3)}}$$

[IN GENERAL,

$\frac{d}{dx}\log_a(g(x)) = \frac{g'(x)}{g(x)\ln(a)}$]

31 $y = x^x = e^{x\ln(x)}$

$$\boxed{y' = e^{x\ln(x)}\left(\ln(x) + 1\right)} = x^x\left(\ln(x)+1\right)$$

or $\longrightarrow$ $\ln(y) = \ln(x^x) = x\ln(x)$

$\frac{d}{dx}$

$$\frac{1}{y}y' = \ln(x) + 1$$

SAME!

$$\therefore\ y' = y\left(\ln(x)+1\right) = x^x\left(\ln(x)+1\right)$$

36 $y = x^{\ln x} = e^{\ln(x)^2}$

$$y' = e^{\ln(x)^2}\cdot 2\ln(x)\cdot\frac{1}{x} = x^{\ln(x)}\cdot 2\ln(x)\cdot\frac{1}{x}$$

$$\boxed{y' = 2x^{\ln(x)-1}\ln(x)}$$

42 $\int_0^1 4^{-2u}\,du$

Let $w = -2u$ | $u = 0 \Rightarrow w = -2(0) = 0$

$dw = -2\,du$ | $u = 1 \Rightarrow w = -2(1) = -2$

$-\frac{1}{2}dw = du$

$$-\frac{1}{2}\int_0^{-2} 4^w\,dw = -\frac{1}{2}\frac{4^w}{\ln(4)}\Bigg|_0^{-2} = \frac{1}{2\ln(4)} - \frac{1}{32\ln(4)}$$

$$= \boxed{\frac{15}{32\ln(4)}}$$

44 $\int (x^5 + 5^x)\,dx = \boxed{\frac{1}{6}x^6 + \frac{5^x}{\ln(5)} + C}$

46 $\int \frac{2^x}{2^x + 1}\,dx$

LET $u = 2^x + 1$

$du = 2^x \ln(2)\,dx$

$\therefore \frac{1}{\ln(2)}\,du = 2^x\,dx$

$$\frac{1}{\ln(2)} \int \frac{1}{u}\,du = \frac{1}{\ln(2)} \ln|u| + C$$

$$= \boxed{\frac{\ln|2^x + 1|}{\ln(2)} + C}$$

NOTE THAT THE ABS. VALUE SIGN IS NOT DOING ANYTHING.