MATH 202 HW #3

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§5.3

2 (a) $e^{\ln(6)} = \boxed{6}$ (b) $\ln\sqrt{e} = \ln(e^{1/2}) = \frac{1}{2}\ln(e) = \boxed{\frac{1}{2}}$

3 (a) $\ln(e^{\sqrt{2}}) = \sqrt{2}\ln(e) = \boxed{\sqrt{2}}$

(b) $e^{3\ln(2)} = (e^{\ln(2)})^3 = 2^3 = \boxed{8}$

4 (a) $\ln(e^{\sin(x)}) = \sin(x)\ln(e) = \boxed{\sin(x)}$

(b) $e^{x+\ln(x)} = e^x e^{\ln(x)} = e^x x$ or $\boxed{xe^x}$

5 (a) $2\ln(x) = 1 \longrightarrow \ln(x) = \frac{1}{2} \longrightarrow x = e^{1/2} = \boxed{\sqrt{e}}$

(b) $e^{-x} = 5 \longrightarrow \ln(e^{-x}) = \ln(5)$

$-x\ln(e) = \ln(5)$

$x = \boxed{-\ln(5)}$

7 (a) $2^{x-5} = 3 \longrightarrow \ln(2^{x-5}) = \ln(3)$

$(x-5)\ln(2) = \ln(3)$

$$\boxed{x = \frac{\ln(3)}{\ln(2)} + 5}$$

(b) $\ln(x) + \ln(x-1) = 1$

$$\ln[x(x-1)] = 1 \longrightarrow e^{\ln[x(x-1)]} = e^{1}$$

$$x(x-1) = e \longrightarrow x^2 - x - e = 0$$

$$x = \frac{1 \pm \sqrt{1+4e}}{2} = \frac{1}{2} \pm \frac{\sqrt{1+4e}}{2}$$

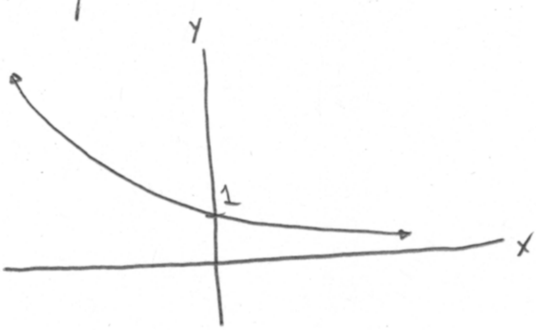
REJECT NEGATIVE ANSWER

BECAUSE THIS WOULD CAUSE $\text{Log}(x-1)$

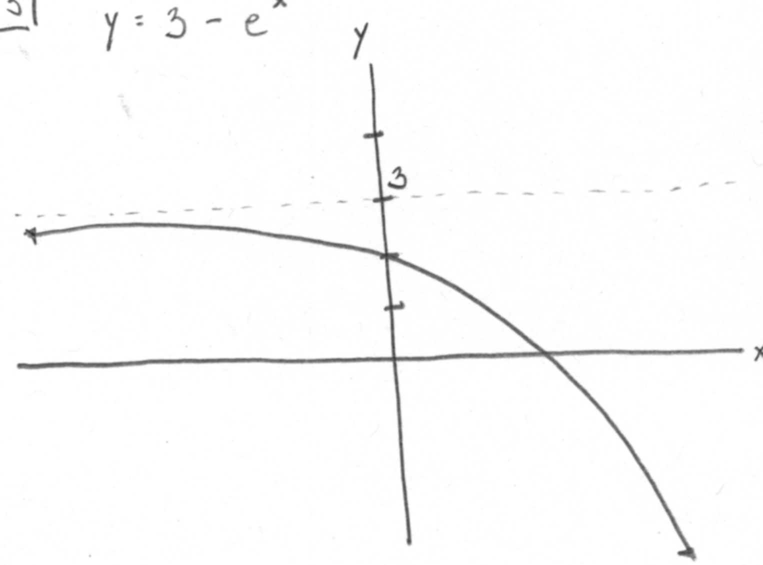
TO BE UNDEFINED!

$$\therefore \boxed{x = \frac{1}{2} + \frac{\sqrt{1+4e}}{2}}$$

$\boxed{11}$ $y = e^{-x}$



$\boxed{13}$ $y = 3 - e^{x}$



15 $$\lim_{x\to\infty} e^{1-x^3} \xrightarrow[\text{SINCE } 1-x^3 \to -\infty \text{ AS } x \to \infty]{} = \lim_{t\to -\infty} e^t = \boxed{0}$$

17 $$\lim_{x\to\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x\to\infty} \frac{1 - e^{-6x}}{1 + e^{-6x}} = \frac{1-0}{1+0} = \boxed{1}$$

19 $$\lim_{x\to 2^+} e^{3/(2-x)} \xrightarrow[\text{SINCE } \frac{3}{2-x} \to -\infty \text{ AS } x \to 2^+]{} = \lim_{t\to -\infty} e^t = \boxed{0}$$

21 $f(x) = x^2 e^x$

$f'(x) = \boxed{2xe^x + x^2 e^x}$

23 $y = e^{ax^3}$

$y' = \boxed{e^{ax^3}(3ax^2)}$

26 $y = e^x \operatorname{Ln}(x)$

$y' = \boxed{e^x \operatorname{Ln}(x) + e^x \cdot \frac{1}{x}}$

31 $y = e^{e^x}$

$y' = e^{e^x}(e^x) = \boxed{e^{e^x + x}}$

37 $$e^{x^2 y} = x + y \xrightarrow{\frac{d}{dx}} e^{x^2 y}(2xy + x^2 y') = 1 + y'$$

$$e^{x^2 y} 2xy + e^{x^2 y} x^2 y' = 1 + y'$$

$$y'(x^2 e^{x^2 y} - 1) = 1 - 2xy e^{x^2 y}$$

"IMPLICIT DIFFERENTIATION"

$$\boxed{y' = \frac{1 - 2xy e^{x^2 y}}{x^2 e^{x^2 y} - 1}}$$

57 $\displaystyle\int_0^5 e^{-3x}\,dx$

LET $u = -3x$

$du = -3\,dx$

$\therefore\ -\frac{1}{3}\,du = dx$

$x = 0 \Rightarrow u = -3(0) = 0$

$x = 5 \Rightarrow u = -3(5) = -15$

$$= -\frac{1}{3}\int_0^{-15} e^u\,du = -\frac{1}{3}e^u\Big|_0^{-15} = \frac{-1}{3e^{15}} + \frac{1}{3} = \frac{e^{15}-1}{3e^{15}}$$

or $\boxed{\frac{1}{3}\left(1 - e^{-15}\right)}$

59 $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

LET $u = 1 + e^x$

$du = e^x\,dx$

$$= \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C \longrightarrow \boxed{\frac{2}{3}(1+e^x)^{3/2} + C}$$

61 $\displaystyle\int \frac{e^x+1}{e^x}\,dx = \int 1 + e^{-x}\,dx$

LET $u = -x$

$du = -dx \Rightarrow -du = dx$

$$-\int 1 + e^u\,du = -(u + e^u) + C$$

$$= \boxed{x - e^{-x} + C}$$