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MATH 202 HW #2

§5.2 # 1, 4, 7 - 11, 13, 19, 25, 31, 38, 51, 54, 56, 57, 60, 61

[1] $\ln \frac{x^3 y}{z^2} = \ln(x^3) + \ln(y) - \ln(z^2)$

$= \boxed{3\ln(x) + \ln(y) - 2\ln(z)}$

[4] $\ln\left(\frac{3x^2}{(x+1)^5}\right) = \ln(3x^2) - \ln\left[(x+1)^5\right]$

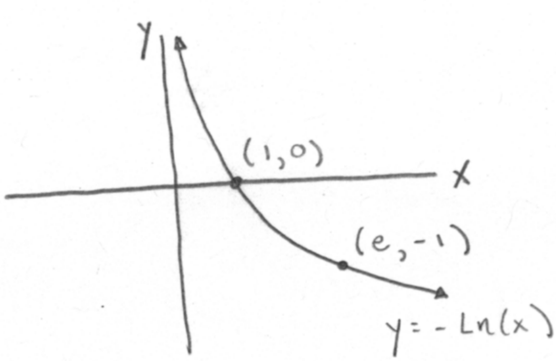
$= \boxed{\ln(3) + 2\ln(x) - 5\ln(x+1)}$

[7] $\frac{1}{2}\ln(x) - 5\ln(x^2+1) = \ln(\sqrt{x}) - \ln\left[(x^2+1)^5\right]$

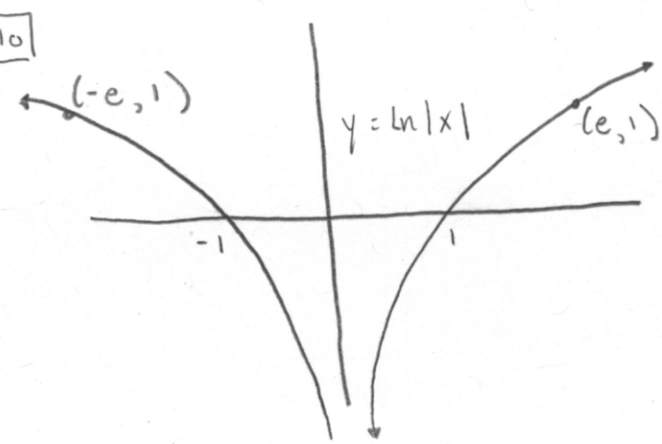
$= \boxed{\ln\left(\frac{\sqrt{x}}{(x^2+1)^5}\right)}$

[8] $\ln(x) + a\ln(y) - b\ln(z) = \boxed{\ln\left(\frac{x y^a}{z^b}\right)}$

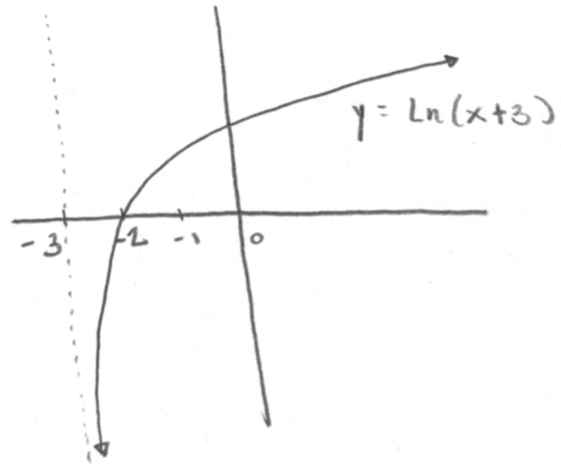
[9]




[10]



11




13

$$f(x) = \sqrt{x}\,\mathrm{Ln}(x)$$

$$f'(x) = \frac{1}{2\sqrt{x}}\mathrm{Ln}(x) + \frac{\sqrt{x}}{x}$$

$$= \frac{\mathrm{Ln}(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}} = \boxed{\frac{\mathrm{Ln}(x)+2}{2\sqrt{x}}}$$

19

$$g(x) = \mathrm{Ln}\left(\frac{a-x}{a+x}\right) = \mathrm{Ln}(a-x) - \mathrm{Ln}(a+x)$$

$$g'(x) = \frac{-1}{a-x} - \frac{1}{a+x} = \frac{-(a+x)-(a-x)}{(a-x)(a+x)} = \boxed{\frac{-2a}{a^2-x^2}}$$

25

$$y = \mathrm{Ln}\,|2-x-5x^2|$$

$$y' = \frac{1}{2-x-5x^2}(-1-10x) = \boxed{\frac{10x+1}{5x^2+x-2}}$$

31

$$y = \mathrm{Ln}(\mathrm{Ln}(x))$$

$$y' = \frac{1}{\mathrm{Ln}(x)}\cdot\frac{1}{x} = \boxed{\frac{1}{x\,\mathrm{Ln}(x)}} \quad \left( = [x\,\mathrm{Ln}(x)]^{-1} \right)$$

$$y'' = -[x\,\mathrm{Ln}(x)]^{-2}\left(\mathrm{Ln}(x) + x\cdot\frac{1}{x}\right)$$

$$= \boxed{-\frac{\mathrm{Ln}(x)+1}{x^2\,\mathrm{Ln}(x)^2}}$$

POINT

SLOPE

$\boxed{38}$ $y = \ln(x^3 - 7)$ @ $(2, 0)$

$$y' = \frac{1}{x^3 - 7}(3x^2) \Rightarrow y'(2) = \frac{3(2)^2}{(2)^3 - 7} = \frac{12}{1} = 12$$

$$\therefore \quad y - 0 = 12(x - 2) = 12x - 24$$

$$\boxed{y = 12x - 24}$$

$\boxed{51}$ $y = (2x+1)^5 (x^4 - 3)^6$

$$\ln(y) = 5\ln(2x+1) + 6\ln(x^4 - 3)$$

$$\Rightarrow \frac{1}{y} y' = 5 \cdot \frac{2}{2x+1} + 6 \cdot \frac{4x^3}{x^4 - 3} = \frac{10}{2x+1} + \frac{24x^3}{x^4 - 3}$$

$$\Rightarrow y' = y\left(\frac{10}{2x+1} + \frac{24x^3}{x^4 - 3}\right) = \boxed{(2x+1)^5 (x^4-3)^6 \left(\frac{10}{2x+1} + \frac{24x^3}{x^4 - 3}\right)}$$

$$y' = 10(2x+1)^4 (x^4-3)^6 + 24x^3 (2x+1)^5 (x^4-3)^5$$

$$y' = 2(2x+1)^4 (x^4-3)^5 \left[5(x^4-3) + 12x^3(2x+1)\right]$$

$$5x^4 - 15 + 24x^4 + 12x^3$$

EITHER ONE IS FINE

$$\boxed{y' = 2(2x+1)^4 (x^4-3)^5 (29x^4 + 12x^3 - 15)}$$

54

$$y = \left(\frac{x^2+1}{x^2-1}\right)^{1/4} = \left(\frac{x^2+1}{(x+1)(x-1)}\right)^{1/4}$$

$$\text{Ln}(y) = \frac{1}{4}\left[\text{Ln}(x^2+1) - \text{Ln}(x+1) - \text{Ln}(x-1)\right]$$

$$\frac{1}{y}y' = \frac{1}{4}\left[\frac{2x}{x^2+1} - \frac{1}{x+1} - \frac{1}{x-1}\right]$$

$$\therefore \boxed{y' = \frac{1}{4}\underbrace{\sqrt[4]{\frac{x^2+1}{x^2-1}}}_{y}\left(\frac{2x}{x^2+1} - \frac{1}{x+1} - \frac{1}{x-1}\right)}$$

56

$$\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 4u^{-3} + \frac{1}{u}\,du = -2u^{-2} + \text{Ln}|u|\Big|_1^2$$

$$= \left(\frac{-2}{2^2} + \text{Ln}|2|\right) - \left(\frac{-2}{1^2} + \text{Ln}|1|\right) = -\frac{1}{2} + \text{Ln}|2| + 2$$

$$= \boxed{\frac{3}{2} + \text{Ln}(2)}$$

57

$$\int_1^e \frac{x^2+x+1}{x}\,dx = \int_1^e x + 1 + \frac{1}{x} = \frac{1}{2}x^2 + x + \text{Ln}|x|\Big|_1^e$$

$$= \left(\frac{e^2}{2} + e + \text{Ln}|e|\right) - \left(\frac{1}{2} + 1 + \text{Ln}|1|\right)$$

$$= \frac{e^2}{2} + e + 1 - \frac{3}{2} = \boxed{\frac{e^2}{2} + e - \frac{1}{2}}$$

60 $\displaystyle\int_e^6 \frac{dx}{x\,Ln(x)}$ LET $u = Ln(x)$

$du = \frac{1}{x}\,dx$

NOTE: $x = e \Rightarrow u = Ln(e) = 1$

$x = 6 \Rightarrow u = Ln(6)$

$$\int_1^{Ln(6)} \frac{1}{u}\,du = \ln u \Big|_1^{Ln(6)} = Ln(Ln\,6) - \underbrace{Ln(1)}_{0}$$

$$= \boxed{Ln(Ln\,6)}$$

61 $\displaystyle\int \frac{(Ln(x))^2}{x}\,dx$ LET $u = Ln(x)$

$du = \frac{1}{x}\,dx$

$$\int u^2\,du = \frac{1}{3}u^3 \longrightarrow \boxed{\frac{1}{3}Ln(x)^3}$$