JOHN ADAMSKI
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MATH 202 HW #1

§5.1 # 3-19, 21-26, 29-32, 35-40

3 $\boxed{\text{NOT } 1\text{-}1}$; $f(2) = f(6)$

4 $\boxed{1\text{-}1}$

5 $\boxed{1\text{-}1}$; PASSES HORIZONTILE LINE TEST

6 $\boxed{\text{NOT } 1\text{-}1}$; 3 ZEROS (ROOTS)

7 $\boxed{\text{NOT } 1\text{-}1}$; FAILS HORIZONTILE LINE TEST

8 $\boxed{1\text{-}1}$

9 $f(x) = x^2 - 2x = x(x-2)$

$\Rightarrow f(0) = f(2) = 0$

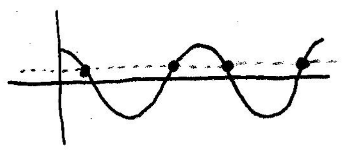
$\therefore \boxed{\text{NOT } 1\text{-}1}$

10 $f(x) = 10 - 3x$

$\boxed{1\text{-}1}$

11 $g(x) = \frac{1}{x}$ $\boxed{1\text{-}1}$

12 $g(x) = \cos x$ $\boxed{\text{NOT } 1\text{-}1}$



(FAILS HORIZONTILE LINE TEST)

e.g. $\cos(0) = \cos(2\pi) = \cos(4\pi) = \ldots$

13 $\boxed{\text{NOT } 1\text{-}1}$

(WHAT GOES UP MUST COME DOWN)

14 $\boxed{\text{NOT 1-1}}$; YOUR HEIGHT AT AGE 33 IS PROBLY THE SAME AS YOUR HEIGHT AT AGE 32 $\left(f(33) = f(32)\right)$.

15 $f(2) = 9$, f IS 1-1 $\Rightarrow$ $\boxed{f^{-1}(9) = 2}$

16 SINCE $f(0) = 0 + \cos(0) = 1$

WE HAVE $\boxed{f^{-1}(1) = 0.}$

[HERE I HAVE ASSUMED f IS 1-1. IN FACT, IT IS! I KNOW THIS BECAUSE $f'(x) = 1 - \sin x \geq 0$ FOR ALL x, AND THE ZEROS OF f' ARE "ISOLATED". HENCE f IS A CONTINUOUS, INCREASING FUNCTION, THUS 1-1.]

17 SINCE $h(4) = 6$, WE CONCLUDE $\boxed{h^{-1}(6) = 4.}$

(NOTE: DOMAIN OF h IS $x \geq 0$)

18 (a) IT DOES NOT TAKE THE SAME VALUE TWICE.

(b) DOMAIN: $[-3, 3]$, RANGE: $[-1, 3]$ (SWITCH DOMAIN & RANGE)

(c) $\boxed{f^{-1}(2) = 0}$

(d) $\boxed{f^{-1}(0) \approx -1\frac{3}{4}}$

19 $C = \frac{5}{9}(F - 32) \Rightarrow$ $\boxed{F = \frac{9}{5}C + 32}$

CONVERTS CELSIUS TO FAHRENHEIT

$F \geq -459.67 \Rightarrow \frac{9}{5}C + 32 \geq -459.67$

$C \geq \frac{5}{9}(-491.67) \Rightarrow \boxed{C \geq -273.15}$

21 $f(x) = 3-2x \longrightarrow y = 3-2x \longrightarrow x = \dfrac{3-y}{2}$ "SOLVE FOR x"

$\therefore \boxed{f^{-1}(x) = \dfrac{3-x}{2}}$

22 $f(x) = \dfrac{4x-1}{2x+3} \longrightarrow y = \dfrac{4x-1}{2x+3} \longrightarrow y(2x+3) = 4x-1$

$\longrightarrow 2xy + 3y = 4x - 1 \longrightarrow 2xy - 4x = -3y - 1$

$\longrightarrow x(2y-4) = -3y-1 \longrightarrow x = \dfrac{-3y-1}{2y-4}$

$\therefore \boxed{f^{-1}(x) = \dfrac{3y+1}{4-2y}}$

23 $y = \sqrt{10-3x} \longrightarrow y^2 = 10-3x \longrightarrow \dfrac{10-y^2}{3} = x$

$\therefore \boxed{f^{-1}(x) = \dfrac{10-x^2}{3}}$

24 $y = 2x^3 + 3 \longrightarrow x^3 = \dfrac{y-3}{2} \longrightarrow x = \left(\dfrac{y-3}{2}\right)^{1/3}$

$\therefore \boxed{f^{-1}(x) = \left(\dfrac{x-3}{2}\right)^{1/3}}$

25 $y = \dfrac{1-\sqrt{x}}{1+\sqrt{x}} \longrightarrow y + y\sqrt{x} = 1-\sqrt{x} \longrightarrow y\sqrt{x} + \sqrt{x} = 1-y$

$\sqrt{x}(y+1) = 1-y \longrightarrow \sqrt{x} = \dfrac{1-y}{1+y} \longrightarrow x = \left(\dfrac{1-y}{1+y}\right)^2$

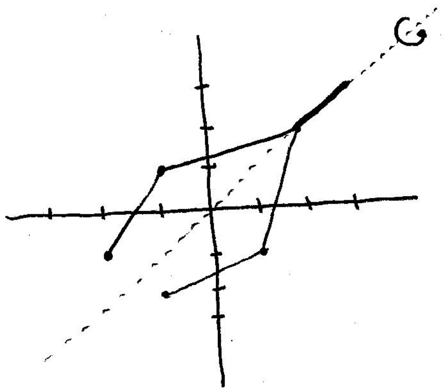
$\therefore \boxed{f^{-1}(x) = \left(\dfrac{1-x}{1+x}\right)^2}$

26 $y = 2x^2 - 8x \longrightarrow \frac{1}{2}y = x^2 - 4x \longrightarrow \frac{1}{2}y + 4 = x^2 - 4x + 4$

$\longrightarrow \frac{1}{2}y + 4 = (x-2)^2 \longrightarrow x - 2 = \sqrt{\frac{1}{2}y + 4}$

$\therefore \boxed{f^{-1}(x) = 2 + \sqrt{\frac{1}{2}x + 4}, \quad x \geq -8}$

29



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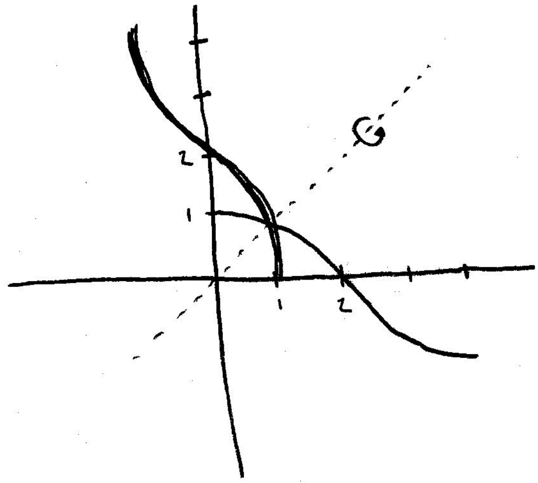


31 (a) SUPPOSE $f(a) = f(b)$. THEN $a^3 = b^3 \Rightarrow (a^3)^{1/3} = (b^3)^{1/3} \Rightarrow a = b$

$\therefore$ f IS 1-1.

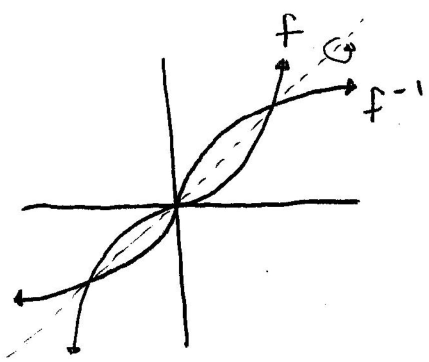
(b) NOTE: $f(2) = 2^3 = 8$ SO $f^{-1}(8) = 2$.

$(f^{-1})'(8) = \frac{1}{f'(2)} = \frac{1}{3(2)^2} = \boxed{\frac{1}{12}}$

(c) $\boxed{f^{-1}(x) = x^{1/3}, \quad \text{DOMAIN}: \mathbb{R}, \quad \text{RANGE}: \mathbb{R}}$

(d) $(f^{-1})'(x) = \frac{1}{3}x^{-2/3} \Rightarrow (f^{-1})'(8) = \frac{1}{3(8)^{2/3}} = \boxed{\frac{1}{12} \checkmark}$

(e)



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(a)

32 SUPPOSE $f(a) = f(b)$. THEN $\sqrt{a-2} = \sqrt{b-2} \Rightarrow a-2 = b-2 \Rightarrow a = b$

$\therefore f$ IS 1-1

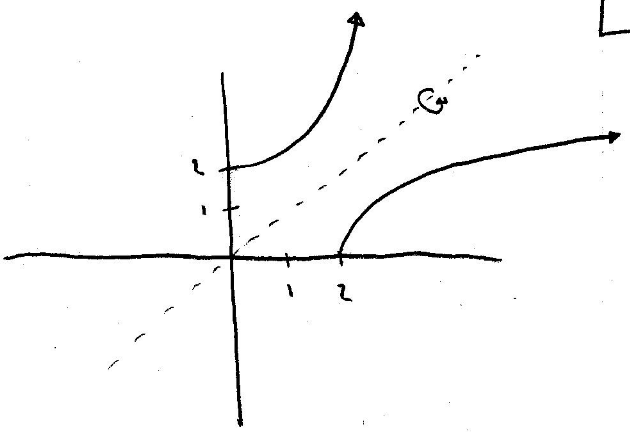
(b) SINCE $f(6) = 2$ WE HAVE

$$(f^{-1})'(2) = \frac{1}{f'(6)} = 2\sqrt{6-2} = \boxed{4}$$

(c) $\boxed{f^{-1}(x) = x^2 + 2 \quad \text{DOMAIN}: [0,\infty) \quad \text{RANGE}: [2,\infty)}$

(d) $(f^{-1})'(x) = 2x \Rightarrow (f^{-1})'(2) = \boxed{4 \quad \checkmark}$

(e)



35 $f(x) = x^3 + x + 1 = 1 \Rightarrow x(x^2+1) = 0 \Rightarrow x = 0$

$$\therefore (f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{3(0)^2+1} = \boxed{1}$$

36 $f(x) = x^5 - x^3 + 2x = 2 \Rightarrow x = 1$

$$\therefore (f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{5(1)^4 - 3(1)^2 + 2} = \boxed{\frac{1}{4}}$$

37 $f(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right) = 3 \quad ; \quad -1 < x < 1$

$$\Rightarrow x^2 + \tan\left(\frac{\pi x}{2}\right) = 0 \quad \Rightarrow \quad x = 0$$

$$\therefore (f^{-1})'(3) = \frac{1}{f'(0)} = \frac{1}{2(0) + \sec^2(0) \cdot \frac{\pi}{2}} = \boxed{\frac{2}{\pi}}$$

38 $f(x) = \sqrt{x^3 + x^2 + x + 1} = 2 \quad \Rightarrow \quad x^3 + x^2 + x + 1 = 4 \quad \Rightarrow \quad x = 1$

$$\therefore (f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{\frac{(3(1)^2 + 2(1) + 1)}{2\sqrt{(1)^3 + (1)^2 + (1) + 1}}}$$

$$= 4/6 = \cancel{2/3} \quad \boxed{\frac{2}{3}}$$

39 $(f^{-1})'(5) = \frac{1}{f'(4)} = \left(\frac{2}{3}\right)^{-1} = \boxed{\frac{3}{2}}$

40 $G(x) = \left[f^{-1}(x)\right]^{-1} \Rightarrow G'(x) = -\left[f^{-1}(x)\right]^{-2} (f^{-1})'(x)$

$$\therefore G'(2) = \frac{-(f^{-1})'(2)}{\left[f^{-1}(2)\right]^2} = \frac{-1/f'(3)}{3^2} = \frac{-9}{9} = \boxed{1}$$