## 5.6 EXERCISES

**1–6** ▪ Find the exact value of each expression.

**1.** (a) $\sin^{-1}(\sqrt{3}/2)$ (b) $\cos^{-1}(-1)$

**2.** (a) $\arctan(-1)$ (b) $\csc^{-1} 2$

**3.** (a) $\tan^{-1}\sqrt{3}$ (b) $\arcsin(-1/\sqrt{2})$

**4.** (a) $\sec^{-1}\sqrt{2}$ (b) $\arcsin 1$

**5.** (a) $\sin(\sin^{-1}(0.7))$ (b) $\tan^{-1}\left(\tan\dfrac{4\pi}{3}\right)$

**6.** (a) $\sec(\arctan 2)$ (b) $\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right)$

**7.** Prove that $\cos(\sin^{-1}x) = \sqrt{1-x^2}$.

**8–10** ▪ Simplify the expression.

**8.** $\tan(\sin^{-1}x)$

**9.** $\sin(\tan^{-1}x)$ **10.** $\csc(\arctan 2x)$

**11.** Prove Formula 6 for the derivative of $\cos^{-1}$ by the same method as for Formula 3.

**12.** (a) Prove that $\sin^{-1}x + \cos^{-1}x = \pi/2$.
(b) Use part (a) to prove Formula 6.

**13.** Prove that $\dfrac{d}{dx}(\cot^{-1}x) = -\dfrac{1}{1+x^2}$.

**14.** Prove that $\dfrac{d}{dx}(\sec^{-1}x) = \dfrac{1}{x\sqrt{x^2-1}}$.

**15.** Prove that $\dfrac{d}{dx}(\csc^{-1}x) = -\dfrac{1}{x\sqrt{x^2-1}}$.

**16–29** ▪ Find the derivative of the function. Simplify where possible.

**16.** $y = \sqrt{\tan^{-1}x}$

**17.** $y = \tan^{-1}\sqrt{x}$ **18.** $h(x) = \sqrt{1-x^2}\arcsin x$

**19.** $y = \sin^{-1}(2x+1)$ **20.** $f(x) = x\ln(\arctan x)$

**21.** $H(x) = (1+x^2)\arctan x$

**22.** $h(t) = e^{\sec^{-1}t}$

**23.** $y = \cos^{-1}(e^{2x})$

**24.** $y = x\cos^{-1}x - \sqrt{1-x^2}$

**25.** $y = \arctan(\cos\theta)$

**26.** $y = \tan^{-1}(x - \sqrt{1+x^2})$

**27.** $h(t) = \cot^{-1}(t) + \cot^{-1}(1/t)$

**28.** $y = \tan^{-1}\left(\dfrac{x}{a}\right) + \ln\sqrt{\dfrac{x-a}{x+a}}$

**29.** $y = \arccos\left(\dfrac{b + a\cos x}{a + b\cos x}\right), \quad 0 \leqslant x \leqslant \pi,\ a > b > 0$

**30–31** ▪ Find the derivative of the function. Find the domains of the function and its derivative.

**30.** $f(x) = \arcsin(e^x)$ **31.** $g(x) = \cos^{-1}(3-2x)$

**32.** Find $y'$ if $\tan^{-1}(xy) = 1 + x^2y$.

**33.** If $g(x) = x\sin^{-1}(x/4) + \sqrt{16-x^2}$, find $g'(2)$.

**34.** Find an equation of the tangent line to the curve $y = 3\arccos(x/2)$ at the point $(1, \pi)$.

**35–38** ▪ Find the limit.

**35.** $\lim_{x\to -1^+}\sin^{-1}x$ **36.** $\lim_{x\to\infty}\arccos\left(\dfrac{1+x^2}{1+2x^2}\right)$

**37.** $\lim_{x\to\infty}\arctan(e^x)$ **38.** $\lim_{x\to 0^+}\tan^{-1}(\ln x)$

**39.** A ladder 10 ft long leans against a vertical wall. If the bottom of the ladder slides away from the base of the wall at a speed of 2 ft/s, how fast is the angle between the ladder and the wall changing when the bottom of the ladder is 6 ft from the base of the wall?

**40.** A lighthouse is located on a small island, 3 km away from the nearest point $P$ on a straight shoreline, and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from $P$?

**41.** Some authors define $y = \sec^{-1}x \iff \sec y = x$ and $y \in [0, \pi/2) \cup (\pi/2, \pi]$. Show that with this definition, we have (instead of the formula given in Exercise 14)

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad |x| > 1$$

**42.** (a) Sketch the graph of the function $f(x) = \sin(\sin^{-1}x)$.
(b) Sketch the graph of the function $g(x) = \sin^{-1}(\sin x)$, $x \in \mathbb{R}$.
(c) Show that $g'(x) = \dfrac{\cos x}{|\cos x|}$.
(d) Sketch the graph of $h(x) = \cos^{-1}(\sin x)$, $x \in \mathbb{R}$, and find its derivative.